

ctrsen.f(3)

LAPACK

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NAME

ctrsen.f –

SYNOPSIS**Functions/Subroutines**

subroutine **ctrsen** (JOB, COMPQ, SELECT, N, T, LDT, Q, LDQ, W, M, S, SEP, WORK, LWORK, INFO)
CTRSEN

Function/Subroutine Documentation

subroutine **ctrsen** (characterJOB, characterCOMPQ, logical, dimension(*)SELECT, integerN, complex, dimension(ldt, *)T, integerLDT, complex, dimension(ldq, *)Q, integerLDQ, complex, dimension(*)W, integerM, realS, realSEP, complex, dimension(*)WORK, integerLWORK, integerINFO)
CTRSEN

Purpose:

CTRSEN reorders the Schur factorization of a complex matrix $A = Q^*TQ^{**}H$, so that a selected cluster of eigenvalues appears in the leading positions on the diagonal of the upper triangular matrix T, and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

Optionally the routine computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

Parameters:*JOB*

JOB is CHARACTER*1

Specifies whether condition numbers are required for the cluster of eigenvalues (S) or the invariant subspace (SEP):

= 'N': none;

= 'E': for eigenvalues only (S);

= 'V': for invariant subspace only (SEP);

= 'B': for both eigenvalues and invariant subspace (S and SEP).

COMPQ

COMPQ is CHARACTER*1

= 'V': update the matrix Q of Schur vectors;

= 'N': do not update Q.

SELECT

SELECT is LOGICAL array, dimension (N)

SELECT specifies the eigenvalues in the selected cluster. To select the j-th eigenvalue, SELECT(j) must be set to .TRUE..

N

N is INTEGER

The order of the matrix T. $N \geq 0$.

T

T is COMPLEX array, dimension (LDT,N)

On entry, the upper triangular matrix T.

On exit, T is overwritten by the reordered matrix T, with the selected eigenvalues as the leading diagonal elements.

LDT

LDT is INTEGER

The leading dimension of the array T. $LDT \geq \max(1,N)$.



Q

Q is COMPLEX array, dimension (LDQ,N)
 On entry, if COMPQ = 'V', the matrix *Q* of Schur vectors.
 On exit, if COMPQ = 'V', *Q* has been postmultiplied by the unitary transformation matrix which reorders *T*; the leading *M* columns of *Q* form an orthonormal basis for the specified invariant subspace.
 If COMPQ = 'N', *Q* is not referenced.

LDQ

LDQ is INTEGER
 The leading dimension of the array *Q*.
LDQ >= 1; and if COMPQ = 'V', *LDQ* >= *N*.

W

W is COMPLEX array, dimension (*N*)
 The reordered eigenvalues of *T*, in the same order as they appear on the diagonal of *T*.

M

M is INTEGER
 The dimension of the specified invariant subspace.
 0 <= *M* <= *N*.

S

S is REAL
 If JOB = 'E' or 'B', *S* is a lower bound on the reciprocal condition number for the selected cluster of eigenvalues.
S cannot underestimate the true reciprocal condition number by more than a factor of sqrt(*N*). If *M* = 0 or *N*, *S* = 1.
 If JOB = 'N' or 'V', *S* is not referenced.

SEP

SEP is REAL
 If JOB = 'V' or 'B', *SEP* is the estimated reciprocal condition number of the specified invariant subspace. If *M* = 0 or *N*, *SEP* = norm(*T*).
 If JOB = 'N' or 'E', *SEP* is not referenced.

WORK

WORK is COMPLEX array, dimension (MAX(1,LWORK))
 On exit, if INFO = 0, *WORK*(1) returns the optimal LWORK.

LWORK

LWORK is INTEGER
 The dimension of the array *WORK*.
 If JOB = 'N', *LWORK* >= 1;
 if JOB = 'E', *LWORK* = max(1,M*(*N*-*M*));
 if JOB = 'V' or 'B', *LWORK* >= max(1,2*M*(*N*-*M*)).

If *LWORK* = -1, then a workspace query is assumed; the routine only calculates the optimal size of the *WORK* array, returns this value as the first entry of the *WORK* array, and no error message related to *LWORK* is issued by XERBLA.

INFO

INFO is INTEGER
 = 0: successful exit
 < 0: if *INFO* = -*i*, the *i*-th argument had an illegal value



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Further Details:

CTRSEN first collects the selected eigenvalues by computing a unitary transformation Z to move them to the top left corner of T . In other words, the selected eigenvalues are the eigenvalues of T_{11} in:

$$Z^* H * T * Z = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{matrix} n_1 \\ n_2 \end{matrix}$$

where $N = n_1 + n_2$. The first n_1 columns of Z span the specified invariant subspace of T .

If T has been obtained from the Schur factorization of a matrix $A = Q^* T Q$, then the reordered Schur factorization of A is given by $A = (Q^* Z)^* (Z^* H^* T^* Z) (Q^* Z) H$, and the first n_1 columns of $Q^* Z$ span the corresponding invariant subspace of A .

The reciprocal condition number of the average of the eigenvalues of T_{11} may be returned in S . S lies between 0 (very badly conditioned) and 1 (very well conditioned). It is computed as follows. First we compute R so that

$$P = \begin{pmatrix} I & R \\ 0 & 0 \end{pmatrix} \begin{matrix} n_1 \\ n_2 \end{matrix}$$

is the projector on the invariant subspace associated with T_{11} . R is the solution of the Sylvester equation:

$$T_{11} * R - R * T_{22} = T_{12}.$$

Let $F\text{-norm}(M)$ denote the Frobenius-norm of M and $2\text{-norm}(M)$ denote the two-norm of M . Then S is computed as the lower bound

$$(1 + F\text{-norm}(R)^2)^{-1/2}$$

on the reciprocal of $2\text{-norm}(P)$, the true reciprocal condition number. S cannot underestimate $1 / 2\text{-norm}(P)$ by more than a factor of \sqrt{N} .

An approximate error bound for the computed average of the eigenvalues of T_{11} is

$$EPS * \text{norm}(T) / S$$

where EPS is the machine precision.

The reciprocal condition number of the right invariant subspace



spanned by the first n_1 columns of Z (or of Q^*Z) is returned in SEP . SEP is defined as the separation of T_{11} and T_{22} :

$$\text{sep}(T_{11}, T_{22}) = \sigma_{\min}(C)$$

where $\sigma_{\min}(C)$ is the smallest singular value of the $n_1 \times n_2$ -by- $n_1 \times n_2$ matrix

$$C = \text{kprod}(I(n_2), T_{11}) - \text{kprod}(\text{transpose}(T_{22}), I(n_1))$$

$I(m)$ is an m by m identity matrix, and kprod denotes the Kronecker product. We estimate $\sigma_{\min}(C)$ by the reciprocal of an estimate of the 1-norm of $\text{inverse}(C)$. The true reciprocal 1-norm of $\text{inverse}(C)$ cannot differ from $\sigma_{\min}(C)$ by more than a factor of $\sqrt{n_1 \times n_2}$.

When SEP is small, small changes in T can cause large changes in the invariant subspace. An approximate bound on the maximum angular error in the computed right invariant subspace is

$$EPS * \text{norm}(T) / SEP$$

Definition at line 264 of file ctrsen.f.

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