## NAME

 realGEauxiliary
## SYNOPSIS

## Functions

subroutine sgesc2 (N, A, LDA, RHS, IPIV, JPIV, SCALE)
SGESC2 solves a system of linear equations using the LU factorization with complete pivoting computed by sgetc2.
subroutine sgetc2 (N, A, LDA, IPIV, JPIV, INFO)
SGETC2 computes the LU factorization with complete pivoting of the general n-by-n matrix. real function slange (NORM, M, N, A, LDA, WORK)

SLANGE returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general rectangular matrix.
subroutine slaqge ( $\mathrm{M}, \mathrm{N}, \mathrm{A}$, LDA, R, C, ROWCND, COLCND, AMAX, EQUED)
SLAQGE scales a general rectangular matrix, using row and column scaling factors computed by sgeequ.
subroutine stgex2 (WANTQ, WANTZ, N, A, LDA, B, LDB, Q, LDQ, Z, LDZ, J1, N1, N2, WORK, LWORK, INFO)
STGEX2 swaps adjacent diagonal blocks in an upper (quasi) triangular matrix pair by an orthogonal equivalence transformation.

## Detailed Description

This is the group of real auxiliary functions for GE matrices

```
Function Documentation
    subroutine sgesc2 (integer N, real, dimension( lda, * ) A, integer LDA, real, dimension( * ) RHS,
        integer, dimension( *) IPIV, integer, dimension(*) JPIV, real SCALE)
        SGESC2 solves a system of linear equations using the LU factorization with complete pivoting
        computed by sgetc2.
```


## Purpose:

SGESC2 solves a system of linear equations
A * X = scale* RHS
with a general N -by- N matrix A using the LU factorization with complete pivoting computed by SGETC2.

## Parameters

$N$
N is INTEGER
The order of the matrix A.
A
A is REAL array, dimension (LDA,N)
On entry, the LU part of the factorization of the n-by-n matrix A computed by SGETC2: $\mathrm{A}=\mathrm{P} * \mathrm{~L} * \mathrm{U} * \mathrm{Q}$
LDA
LDA is INTEGER
The leading dimension of the array A. LDA $>=\max (1, \mathrm{~N})$.

RHS is REAL array, dimension (N).
On entry, the right hand side vector b .
On exit, the solution vector X .
IPIV
IPIV is INTEGER array, dimension (N).
The pivot indices; for $1<=\mathrm{i}<=\mathrm{N}$, row i of the
matrix has been interchanged with row IPIV(i).

## JPIV

JPIV is INTEGER array, dimension (N).
The pivot indices; for $1<=\mathrm{j}<=\mathrm{N}$, column j of the matrix has been interchanged with column JPIV(j).
SCALE
SCALE is REAL
On exit, SCALE contains the scale factor. SCALE is chosen $0<=$ SCALE $<=1$ to prevent overflow in the solution.

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December 2016

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subroutine sgetc2 (integer N , real, dimension( lda, * ) A, integer LDA, integer, dimension( *) IPIV, integer, dimension( * ) JPIV, integer INFO)
SGETC2 computes the LU factorization with complete pivoting of the general n-by-n matrix.

## Purpose:

SGETC2 computes an LU factorization with complete pivoting of the n-by-n matrix A. The factorization has the form $\mathrm{A}=\mathrm{P} * \mathrm{~L} * \mathrm{U} * \mathrm{Q}$, where $P$ and $Q$ are permutation matrices, $L$ is lower triangular with unit diagonal elements and $U$ is upper triangular.

This is the Level 2 BLAS algorithm.

## Parameters

$N$
N is INTEGER
The order of the matrix A. $\mathrm{N}>=0$.
A
A is REAL array, dimension (LDA, N)
On entry, the n-by-n matrix A to be factored.
On exit, the factors L and U from the factorization
$\mathrm{A}=\mathrm{P} * \mathrm{~L} * \mathrm{U} * \mathrm{Q}$; the unit diagonal elements of L are not stored. If $U(k, k)$ appears to be less than $\operatorname{SMIN}, \mathrm{U}(\mathrm{k}, \mathrm{k})$ is given the value of SMIN, i.e., giving a nonsingular perturbed system.

LDA
LDA is INTEGER
The leading dimension of the array A. LDA $>=\max (1, \mathrm{~N})$. IPIV

IPIV is INTEGER array, dimension( N ).
The pivot indices; for $1<=\mathrm{i}<=\mathrm{N}$, row i of the matrix has been interchanged with row IPIV(i). JPIV

JPIV is INTEGER array, dimension(N).
The pivot indices; for $1<=\mathrm{j}<=\mathrm{N}$, column j of the
matrix has been interchanged with column $\operatorname{JPIV}(\mathrm{j})$.
INFO
INFO is INTEGER
= 0 : successful exit
$>0$ : if $\mathrm{INFO}=\mathrm{k}, \mathrm{U}(\mathrm{k}, \mathrm{k})$ is likely to produce overflow if we try to solve for x in $\mathrm{Ax}=\mathrm{b}$. So U is perturbed to avoid the overflow.

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real function slange (character NORM, integer $M$, integer $N$, real, dimension( lda, *) A, integer LDA, real, dimension( *) WORK)
SLANGE returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general rectangular matrix.

## Purpose:

SLANGE returns the value of the one norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real matrix A .

## Returns

SLANGE

```
SLANGE = ( max(abs(A(i,j))), NORM = 'M' or 'm'
            (
                ( norm1(A), NORM = '1', 'O' or 'o'
            (
            ( normI(A), NORM = 'I' or 'i'
            (
            ( normF(A), NORM = 'F', 'f', 'E' or 'e'
```

where norm1 denotes the one norm of a matrix (maximum column sum), normI denotes the infinity norm of a matrix (maximum row sum) and normF denotes the Frobenius norm of a matrix (square root of sum of squares). Note that $\max (\mathrm{abs}(\mathrm{A}(\mathrm{i}, \mathrm{j}))$ ) is not a consistent matrix norm.

## Parameters

NORM

## NORM is CHARACTER*1

Specifies the value to be returned in SLANGE as described above.

M
M is INTEGER
The number of rows of the matrix $A . M>=0$. When $M=0$, SLANGE is set to zero.
$N$
N is INTEGER

The number of columns of the matrix $\mathrm{A} . \mathrm{N}>=0$. When $\mathrm{N}=0$, SLANGE is set to zero.

A
A is REAL array, dimension (LDA,N)
The m by n matrix A .
LDA
LDA is INTEGER
The leading dimension of the array $A$. LDA $>=\max (\mathrm{M}, 1)$.
WORK
WORK is REAL array, dimension (MAX(1,LWORK)), where LWORK >= M when NORM = 'I'; otherwise, WORK is not referenced.

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subroutine slaqge (integer $M$, integer $N$, real, dimension( lda, * ) A, integer LDA, real, dimension( *)
R, real, dimension( * ) C, real ROWCND, real COLCND, real AMAX, character EQUED)
SLAQGE scales a general rectangular matrix, using row and column scaling factors computed by sgeequ.

## Purpose:

SLAQGE equilibrates a general M by N matrix A using the row and column scaling factors in the vectors R and C .

## Parameters

M
M is INTEGER
The number of rows of the matrix $\mathrm{A} . \mathrm{M}>=0$.
$N$
N is INTEGER
The number of columns of the matrix $\mathrm{A} . \mathrm{N}>=0$.
A
A is REAL array, dimension (LDA,N)
On entry, the M by N matrix A .
On exit, the equilibrated matrix. See EQUED for the form of the equilibrated matrix.

LDA
LDA is INTEGER
The leading dimension of the array $A$. LDA $>=\max (\mathrm{M}, 1)$.
R
R is REAL array, dimension (M)
The row scale factors for A .
C
C is REAL array, dimension ( N )
The column scale factors for $A$.

## ROWCND

ROWCND is REAL
Ratio of the smallest $\mathrm{R}(\mathrm{i})$ to the largest $\mathrm{R}(\mathrm{i})$.

## COLCND

COLCND is REAL
Ratio of the smallest $\mathrm{C}(\mathrm{i})$ to the largest $\mathrm{C}(\mathrm{i})$.
AMAX
AMAX is REAL
Absolute value of largest matrix entry.
EQUED

## EQUED is CHARACTER*1

Specifies the form of equilibration that was done.
$=$ ' N ': No equilibration
$=$ 'R': Row equilibration, i.e., A has been premultiplied by $\operatorname{diag}(\mathrm{R})$.
$=$ ' C ': Column equilibration, i.e., A has been postmultiplied by $\operatorname{diag}(\mathrm{C})$.
= 'B': Both row and column equilibration, i.e., A has been replaced by $\operatorname{diag}(\mathrm{R}) * \mathrm{~A} * \operatorname{diag}(\mathrm{C})$.

## Internal Parameters:

THRESH is a threshold value used to decide if row or column scaling should be done based on the ratio of the row or column scaling factors. If ROWCND < THRESH, row scaling is done, and if COLCND < THRESH, column scaling is done.

LARGE and SMALL are threshold values used to decide if row scaling should be done based on the absolute size of the largest matrix element. If AMAX > LARGE or AMAX < SMALL, row scaling is done.

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December 2016
subroutine stgex2 (logical WANTQ, logical WANTZ, integer N, real, dimension( lda, *) A, integer LDA, real, dimension( ldb, * ) B, integer LDB, real, dimension( ldq, *) Q, integer LDQ, real, dimension( ldz, *) Z, integer LDZ, integer J1, integer N1, integer N2, real, dimension( *)
WORK, integer LWORK, integer INFO)
STGEX2 swaps adjacent diagonal blocks in an upper (quasi) triangular matrix pair by an orthogonal equivalence transformation.

## Purpose:

STGEX2 swaps adjacent diagonal blocks (A11, B11) and (A22, B22)
of size 1-by-1 or 2-by-2 in an upper (quasi) triangular matrix pair (A, B) by an orthogonal equivalence transformation.
(A, B) must be in generalized real Schur canonical form (as returned by SGGES), i.e. A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. $B$ is upper triangular.

Optionally, the matrices Q and Z of generalized Schur vectors are updated.

$$
\begin{aligned}
& \mathrm{Q}(\mathrm{in}) * \mathrm{~A}(\mathrm{in}) * \mathrm{Z}(\mathrm{in}) * * \mathrm{~T}=\mathrm{Q}(\text { out }) * \mathrm{~A}(\text { out }) * \mathrm{Z}(\text { out }) * * \mathrm{~T} \\
& \mathrm{Q}(\mathrm{in}) * \mathrm{~B}(\mathrm{in}) * \mathrm{Z}(\mathrm{in}) * * \mathrm{~T}=\mathrm{Q}(\text { out }) * \mathrm{~B}(\text { out }) * \mathrm{Z}(\text { out }) * * \mathrm{~T}
\end{aligned}
$$

## Parameters

WANTQ

## WANTQ is LOGICAL

.TRUE. : update the left transformation matrix Q;
.FALSE.: do not update Q .

## WANTZ

WANTZ is LOGICAL
.TRUE. : update the right transformation matrix Z;
.FALSE.: do not update Z .
$N$
N is INTEGER
The order of the matrices A and B. $\mathrm{N}>=0$.
A
A is REAL array, dimension (LDA,N)
On entry, the matrix A in the pair (A, B).
On exit, the updated matrix A.
LDA
LDA is INTEGER
The leading dimension of the array A. LDA $>=\max (1, \mathrm{~N})$.
B
B is REAL array, dimension (LDB,N)
On entry, the matrix $B$ in the pair (A, B).
On exit, the updated matrix B.
LDB
LDB is INTEGER
The leading dimension of the array B. LDB $>=\max (1, \mathrm{~N})$.
$Q$
Q is REAL array, dimension (LDQ,N)
On entry, if WANTQ = .TRUE., the orthogonal matrix Q .
On exit, the updated matrix $Q$.
Not referenced if WANTQ = .FALSE..
$L D Q$
LDQ is INTEGER
The leading dimension of the array $\mathrm{Q} . \mathrm{LDQ}>=1$.
If WANTQ = .TRUE., LDQ >= N .
Z
Z is REAL array, dimension (LDZ,N)
On entry, if WANTZ =.TRUE., the orthogonal matrix Z.
On exit, the updated matrix Z .
Not referenced if WANTZ = .FALSE..
LDZ
LDZ is INTEGER
The leading dimension of the array Z . $\mathrm{LDZ}>=1$.
If WANTZ = .TRUE., LDZ >= N.

J 1 is INTEGER
The index to the first block (A11, B11). $1<=\mathrm{J} 1<=\mathrm{N}$.
N1
N1 is INTEGER
The order of the first block (A11, B11). $\mathrm{N} 1=0,1$ or 2 .
N2
N2 is INTEGER
The order of the second block (A22, B22). N2 = 0, 1 or 2 .
WORK
WORK is REAL array, dimension (MAX (1,LWORK)).
LWORK
LWORK is INTEGER
The dimension of the array WORK.
LWORK > $=\operatorname{MAX}\left(\mathrm{N}^{*}(\mathrm{~N} 2+\mathrm{N} 1),(\mathrm{N} 2+\mathrm{N} 1) *(\mathrm{~N} 2+\mathrm{N} 1) * 2\right)$
INFO

## INFO is INTEGER

$=0$ : Successful exit
$>0$ : If $\mathrm{INFO}=1$, the transformed matrix (A, B) would be too far from generalized Schur form; the blocks are not swapped and $(\mathrm{A}, \mathrm{B})$ and $(\mathrm{Q}, \mathrm{Z})$ are unchanged. The problem of swapping is too ill-conditioned.
$<0$ : If INFO $=-16$ : LWORK is too small. Appropriate value for LWORK is returned in WORK(1).

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## Further Details:

In the current code both weak and strong stability tests are performed. The user can omit the strong stability test by changing the internal logical parameter WANDS to .FALSE.. See ref. [2] for details.

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## References:

[1] B. Kagstrom; A Direct Method for Reordering Eigenvalues in the Generalized Real Schur Form of a Regular Matrix Pair (A, B), in M.S. Moonen et al (eds), Linear Algebra for Large Scale and Real-Time Applications, Kluwer Academic Publ. 1993, pp 195-218.
[2] B. Kagstrom and P. Poromaa; Computing Eigenspaces with Specified Eigenvalues of a Regular Matrix Pair (A, B) and Condition Estimation: Theory, Algorithms and Software, Report UMINF - 94.04, Department of Computing Science, Umea University, S-901 87 Umea, Sweden, 1994. Also as LAPACK Working Note 87. To appear in Numerical Algorithms, 1996.

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