NAME

OTHERauxiliary

SYNOPSIS

Modules

double
real
complex
complex16

Functions

subroutine **dcombssq** (V1, V2)
DCOMBSSQ adds two scaled sum of squares quantities.

logical function **disnan** (DIN)
DISNAN tests input for NaN.

subroutine **dlabad** (SMALL, LARGE)
DLABAD

subroutine **dlacpy** (UPLO, M, N, A, LDA, B, LDB)
DLACPY copies all or part of one two-dimensional array to another.

subroutine **dlae2** (A, B, C, RT1, RT2)
DLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix.

subroutine **dlaebz** (IJOB, NITMAX, N, MMAX, MINP, NBMIN, ABSTOL, RELTOL, PIVMIN, D, E, E2, NVAL, AB, C, MOUT, NAB, WORK, IWORK, INFO)
DLAEBZ computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine sstebz.

subroutine **dlaev2** (A, B, C, RT1, RT2, CS1, SN1)
DLAEV2 computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.

subroutine **dlagts** (JOB, N, A, B, C, D, IN, Y, TOL, INFO)
DLAGTS solves the system of equations (T-λI)x = y or (T-λI)Tx = y,where T is a general tridiagonal matrix and λ a scalar, using the LU factorization computed by slagtf.

logical function **dlaisnan** (DIN1, DIN2)
DLAISNAN tests input for NaN by comparing two arguments for inequality.

integer function **dlaneg** (N, D, LLD, SIGMA, PIVMIN, R)
DLANEG computes the Sturm count.

double precision function **dlanst** (NORM, N, D, E)
DLANST returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix.

double precision function **dlapy2** (X, Y)
DLAPY2 returns sqrt(x2+y2).

double precision function **dlapy3** (X, Y, Z)
DLAPY3 returns sqrt(x2+y2+z2).

subroutine **dlarnv** (IDIST, ISEED, N, X)
DLARNV returns a vector of random numbers from a uniform or normal distribution.

subroutine **dlarra** (N, D, E, E2, SPLTOL, TNRM, NSPLIT, ISPLIT, INFO)
DLARRA computes the splitting points with the specified threshold.

subroutine **dlarrb** (N, D, LLD, IFIRST, ILAST, RTOL1, RTOL2, OFFSET, W, WGAP, WERR, WORK, IWORK, PIVMIN, SPDIAM, TWIST, INFO)
DLARRB provides limited bisection to locate eigenvalues for more accuracy.

subroutine **dlarrc** (JOBT, N, VL, VU, D, E, PIVMIN, EIGCNT, LCNT, RCNT, INFO)
DLARRC computes the number of eigenvalues of the symmetric tridiagonal matrix.

subroutine **dlarrd** (RANGE, ORDER, N, VL, VU, IL, IU, GERS, RELTOL, D, E, E2, PIVMIN, NSPLIT, ISPLIT, M, W, WERR, WL, WU, IBLOCK, INDEXW, WORK, IWORK, INFO)
DLARRD computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

subroutine **dlarre** (RANGE, N, VL, VU, IL, IU, D, E, E2, RTOL1, RTOL2, SPLTOL, NSPLIT, ISPLIT, M, W, WERR, WGAP, IBLOCK, INDEXW, GERS, PIVMIN, WORK, IWORK, INFO)
DLARRE given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block Ti, finds base representations and eigenvalues.

subroutine **dlarrf** (N, D, L, LD, CLSTRT, CLEND, W, WGAP, WERR, SPDIAM, CLGAPL, CLGAPR, PIVMIN, SIGMA, DPLUS, LPLUS, WORK, INFO)



DLARRF finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

subroutine **dlarrj** (N, D, E2, IFIRST, ILAST, RTOL, OFFSET, W, WERR, WORK, IWORK, PIVMIN, SPDIAM, INFO)

DLARRJ performs refinement of the initial estimates of the eigenvalues of the matrix T.

subroutine **dlarrk** (N, IW, GL, GU, D, E2, PIVMIN, RELTOL, W, WERR, INFO)

DLARRK computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.

subroutine **dlarrr** (N, D, E, INFO)

DLARRR performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

subroutine **dlartg** (F, G, CS, SN, R)

DLARTG generates a plane rotation with real cosine and real sine.

subroutine **dlartgp** (F, G, CS, SN, R)

DLARTGP generates a plane rotation so that the diagonal is nonnegative.

subroutine **dlaruv** (ISEED, N, X)

DLARUV returns a vector of n random real numbers from a uniform distribution.

subroutine **dlas2** (F, G, H, SSMIN, SSMAX)

DLAS2 computes singular values of a 2-by-2 triangular matrix.

subroutine **dlascl** (TYPE, KL, KU, CFROM, CTO, M, N, A, LDA, INFO)

DLASCL multiplies a general rectangular matrix by a real scalar defined as cto/cfrom.

subroutine **dlasd0** (N, SQRE, D, E, U, LDU, VT, LDVT, SMLSIZ, IWORK, WORK, INFO)

DLASD0 computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **dlasd1** (NL, NR, SQRE, D, ALPHA, BETA, U, LDU, VT, LDVT, IDXQ, IWORK, WORK, INFO)

DLASD1 computes the SVD of an upper bidiagonal matrix B of the specified size. Used by sbdsdc.

subroutine **dlasd2** (NL, NR, SQRE, K, D, Z, ALPHA, BETA, U, LDU, VT, LDVT, DSIGMA, U2, LDU2, VT2, LDVT2, IDXP, IDX, IDXC, IDXQ, COLTYP, INFO)

DLASD2 merges the two sets of singular values together into a single sorted set. Used by sbdsdc.

subroutine **dlasd3** (NL, NR, SQRE, K, D, Q, LDQ, DSIGMA, U, LDU, U2, LDU2, VT, LDVT, VT2, LDVT2, IDXC, CTOT, Z, INFO)

DLASD3 finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by sbdsdc.

subroutine **dlasd4** (N, I, D, Z, DELTA, RHO, SIGMA, WORK, INFO)

DLASD4 computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by dbdsdc.

subroutine **dlasd5** (I, D, Z, DELTA, RHO, DSIGMA, WORK)

DLASD5 computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix. Used by sbdsdc.

subroutine **dlasd6** (ICOMPQ, NL, NR, SQRE, D, VF, VL, ALPHA, BETA, IDXQ, PERM, GIVPTR, GIVCOL, LDGCOL, GIVNUM, LDGNUM, POLES, DIFL, DIFR, Z, K, C, S, WORK, IWORK, INFO)

DLASD6 computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by sbdsdc.

subroutine **dlasd7** (ICOMPQ, NL, NR, SQRE, K, D, Z, ZW, VF, VFW, VL, VLW, ALPHA, BETA, DSIGMA, IDX, IDXP, IDXQ, PERM, GIVPTR, GIVCOL, LDGCOL, GIVNUM, LDGNUM, C, S, INFO)

DLASD7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by sbdsdc.

subroutine **dlasd8** (ICOMPQ, K, D, Z, VF, VL, DIFL, DIFR, LDDIFR, DSIGMA, WORK, INFO)

DLASD8 finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by sbdsdc.

subroutine **dlasda** (ICOMPQ, SMLSIZ, N, SQRE, D, E, U, LDU, VT, K, DIFL, DIFR, Z, POLES, GIVPTR, GIVCOL, LDGCOL, PERM, GIVNUM, C, S, WORK, IWORK, INFO)

DLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **dlasdq** (UPLO, SQRE, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C, LDC, WORK,



INFO)
DLASDQ computes the SVD of a real bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **dlasdt** (N, LVL, ND, INODE, NDIML, NDIMR, MSUB)
DLASDT creates a tree of subproblems for bidiagonal divide and conquer. Used by sbdsdc.

subroutine **dlaset** (UPLO, M, N, ALPHA, BETA, A, LDA)
DLASET initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

subroutine **dlasr** (SIDE, PIVOT, DIRECT, M, N, C, S, A, LDA)
DLASR applies a sequence of plane rotations to a general rectangular matrix.

subroutine **dlassq** (N, X, INCX, SCALE, SUMSQ)
DLASSQ updates a sum of squares represented in scaled form.

subroutine **dlasv2** (F, G, H, SSMIN, SSMAX, SNR, CSR, SNL, CSL)
DLASV2 computes the singular value decomposition of a 2-by-2 triangular matrix.

integer function **ieeeck** (ISPEC, ZERO, ONE)
IEEECK

integer function **iladlc** (M, N, A, LDA)
ILADLC scans a matrix for its last non-zero column.

integer function **iladlr** (M, N, A, LDA)
ILADLR scans a matrix for its last non-zero row.

integer function **ilaenv** (ISPEC, NAME, OPTS, N1, N2, N3, N4)
ILAENV

integer function **ilaenv2stage** (ISPEC, NAME, OPTS, N1, N2, N3, N4)
ILAENV2STAGE

integer function **iparmq** (ISPEC, NAME, OPTS, N, ILO, IHI, LWORK)
IPARMQ

logical function **lsamen** (N, CA, CB)
LSAMEN

subroutine **scombssq** (V1, V2)
SCOMBSSQ adds two scaled sum of squares quantities

logical function **sisnan** (SIN)
SISNAN tests input for NaN.

subroutine **slabad** (SMALL, LARGE)
SLABAD

subroutine **slacpy** (UPLO, M, N, A, LDA, B, LDB)
SLACPY copies all or part of one two-dimensional array to another.

subroutine **slae2** (A, B, C, RT1, RT2)
SLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix.

subroutine **slaebz** (IJOB, NITMAX, N, MMAX, MINP, NBMIN, ABSTOL, RELTOL, PIVMIN, D, E, E2, NVAL, AB, C, MOUT, NAB, WORK, IWORK, INFO)
SLAEBZ computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine sstebz.

subroutine **slaev2** (A, B, C, RT1, RT2, CS1, SN1)
SLAEV2 computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.

subroutine **slag2d** (M, N, SA, LDSA, A, LDA, INFO)
SLAG2D converts a single precision matrix to a double precision matrix.

subroutine **slagts** (JOB, N, A, B, C, D, IN, Y, TOL, INFO)
SLAGTS solves the system of equations (T-λI)x = y or (T-λI)Tx = y,where T is a general tridiagonal matrix and λ a scalar, using the LU factorization computed by slagtf.

logical function **slaisnan** (SIN1, SIN2)
SLAISNAN tests input for NaN by comparing two arguments for inequality.

integer function **slaneg** (N, D, LLD, SIGMA, PIVMIN, R)
SLANEG computes the Sturm count.

real function **slanst** (NORM, N, D, E)
SLANST returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix.

real function **slapy2** (X, Y)
SLAPY2 returns sqrt(x2+y2).

real function **slapy3** (X, Y, Z)



SLAPY3 returns sqrt(x2+y2+z2).

subroutine **slarnv** (IDIST, ISEED, N, X)

SLARNV returns a vector of random numbers from a uniform or normal distribution.

subroutine **slarra** (N, D, E, E2, SPLTOL, TNRM, NSPLIT, ISPLIT, INFO)

SLARRA computes the splitting points with the specified threshold.

subroutine **slarrb** (N, D, LLD, IFIRST, ILAST, RTOL1, RTOL2, OFFSET, W, WGAP, WERR, WORK, IWORK, PIVMIN, SPDIAM, TWIST, INFO)

SLARRB provides limited bisection to locate eigenvalues for more accuracy.

subroutine **slarrc** (JOBT, N, VL, VU, D, E, PIVMIN, EIGCNT, LCNT, RCNT, INFO)

SLARRC computes the number of eigenvalues of the symmetric tridiagonal matrix.

subroutine **slarrd** (RANGE, ORDER, N, VL, VU, IL, IU, GERS, RELTOL, D, E, E2, PIVMIN, NSPLIT, ISPLIT, M, W, WERR, WL, WU, IBLOCK, INDEXW, WORK, IWORK, INFO)

SLARRD computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

subroutine **slarre** (RANGE, N, VL, VU, IL, IU, D, E, E2, RTOL1, RTOL2, SPLTOL, NSPLIT, ISPLIT, M, W, WERR, WGAP, IBLOCK, INDEXW, GERS, PIVMIN, WORK, IWORK, INFO)

SLARRE given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block Ti, finds base representations and eigenvalues.

subroutine **slarrf** (N, D, L, LD, CLSTRT, CLEND, W, WGAP, WERR, SPDIAM, CLGAPL, CLGAPR, PIVMIN, SIGMA, DPLUS, LPLUS, WORK, INFO)

SLARRF finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

subroutine **slarrj** (N, D, E2, IFIRST, ILAST, RTOL, OFFSET, W, WERR, WORK, IWORK, PIVMIN, SPDIAM, INFO)

SLARRJ performs refinement of the initial estimates of the eigenvalues of the matrix T.

subroutine **slarrk** (N, IW, GL, GU, D, E2, PIVMIN, RELTOL, W, WERR, INFO)

SLARRK computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.

subroutine **slarrr** (N, D, E, INFO)

SLARRR performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

subroutine **slartg** (F, G, CS, SN, R)

SLARTG generates a plane rotation with real cosine and real sine.

subroutine **slartgp** (F, G, CS, SN, R)

SLARTGP generates a plane rotation so that the diagonal is nonnegative.

subroutine **slaruv** (ISEED, N, X)

SLARUV returns a vector of n random real numbers from a uniform distribution.

subroutine **slas2** (F, G, H, SSMIN, SSMAX)

SLAS2 computes singular values of a 2-by-2 triangular matrix.

subroutine **slascl** (TYPE, KL, KU, CFROM, CTO, M, N, A, LDA, INFO)

SLASCL multiplies a general rectangular matrix by a real scalar defined as cto/cfrom.

subroutine **slasd0** (N, SQRE, D, E, U, LDU, VT, LDVT, SMLSIZ, IWORK, WORK, INFO)

SLASD0 computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **slasd1** (NL, NR, SQRE, D, ALPHA, BETA, U, LDU, VT, LDVT, IDXQ, IWORK, WORK, INFO)

SLASD1 computes the SVD of an upper bidiagonal matrix B of the specified size. Used by sbdsdc.

subroutine **slasd2** (NL, NR, SQRE, K, D, Z, ALPHA, BETA, U, LDU, VT, LDVT, DSIGMA, U2, LDU2, VT2, LDVT2, IDXP, IDX, IDXC, IDXQ, COLTYP, INFO)

SLASD2 merges the two sets of singular values together into a single sorted set. Used by sbdsdc.

subroutine **slasd3** (NL, NR, SQRE, K, D, Q, LDQ, DSIGMA, U, LDU, U2, LDU2, VT, LDVT, VT2, LDVT2, IDXC, CTOT, Z, INFO)

SLASD3 finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by sbdsdc.

subroutine **slasd4** (N, I, D, Z, DELTA, RHO, SIGMA, WORK, INFO)

SLASD4 computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by sbdsdc.

subroutine **slasd5** (I, D, Z, DELTA, RHO, DSIGMA, WORK)



SLASD5 computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix. Used by sbdsdc.

subroutine **slasd6** (ICOMPQ, NL, NR, SQRE, D, VF, VL, ALPHA, BETA, IDXQ, PERM, GIVPTR, GIVCOL, LDGCOL, GIVNUM, LDGNUM, POLES, DIFL, DIFR, Z, K, C, S, WORK, IWORK, INFO)

SLASD6 computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by sbdsdc.

subroutine **slasd7** (ICOMPQ, NL, NR, SQRE, K, D, Z, ZW, VF, VFW, VL, VLW, ALPHA, BETA, DSIGMA, IDX, IDXP, IDXQ, PERM, GIVPTR, GIVCOL, LDGCOL, GIVNUM, LDGNUM, C, S, INFO)

SLASD7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by sbdsdc.

subroutine **slasd8** (ICOMPQ, K, D, Z, VF, VL, DIFL, DIFR, LDDIFR, DSIGMA, WORK, INFO)

SLASD8 finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by sbdsdc.

subroutine **slasda** (ICOMPQ, SMLSIZ, N, SQRE, D, E, U, LDU, VT, K, DIFL, DIFR, Z, POLES, GIVPTR, GIVCOL, LDGCOL, PERM, GIVNUM, C, S, WORK, IWORK, INFO)

SLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **slasdq** (UPLO, SQRE, N, NCVT, NRU, NCC, D, E, VT, LDVT, U, LDU, C, LDC, WORK, INFO)

SLASDQ computes the SVD of a real bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

subroutine **slasdt** (N, LVL, ND, INODE, NDIML, NDIMR, MSUB)

SLASDT creates a tree of subproblems for bidiagonal divide and conquer. Used by sbdsdc.

subroutine **slaset** (UPLO, M, N, ALPHA, BETA, A, LDA)

SLASET initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

subroutine **slasr** (SIDE, PIVOT, DIRECT, M, N, C, S, A, LDA)

SLASR applies a sequence of plane rotations to a general rectangular matrix.

subroutine **slassq** (N, X, INCX, SCALE, SUMSQ)

SLASSQ updates a sum of squares represented in scaled form.

subroutine **slasv2** (F, G, H, SSMIN, SSMAX, SNR, CSR, SNL, CSL)

SLASV2 computes the singular value decomposition of a 2-by-2 triangular matrix.

subroutine **xerbla** (SRNAME, INFO)

XERBLA

subroutine **xerbla_array** (SRNAME_ARRAY, SRNAME_LEN, INFO)

XERBLA_ARRAY

Detailed Description

This is the group of Other Auxiliary routines

Function Documentation

subroutine dcombssq (double precision, dimension(2) V1, double precision, dimension(2) V2)

DCOMBSSQ adds two scaled sum of squares quantities.

Purpose:

DCOMBSSQ adds two scaled sum of squares quantities, V1 := V1 + V2.
That is,

```
V1_scale**2 * V1_sumsq := V1_scale**2 * V1_sumsq
                        + V2_scale**2 * V2_sumsq
```

Parameters

V1

V1 is DOUBLE PRECISION array, dimension (2).
The first scaled sum.
V1(1) = V1_scale, V1(2) = V1_sumsq.

V2



V2 is DOUBLE PRECISION array, dimension (2).
The second scaled sum.
V2(1) = V2_scale, V2(2) = V2_sumsq.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

November 2018

logical function disnan (double precision, intent(in) DIN)

DISNAN tests input for NaN.

Purpose:

DISNAN returns .TRUE. if its argument is NaN, and .FALSE.
otherwise. To be replaced by the Fortran 2003 intrinsic in the
future.

Parameters

DIN

DIN is DOUBLE PRECISION
Input to test for NaN.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

subroutine dlabad (double precision SMALL, double precision LARGE)

DLABAD

Purpose:

DLABAD takes as input the values computed by DLAMCH for underflow and
overflow, and returns the square root of each of these values if the
log of LARGE is sufficiently large. This subroutine is intended to
identify machines with a large exponent range, such as the Crays, and
redefine the underflow and overflow limits to be the square roots of
the values computed by DLAMCH. This subroutine is needed because
DLAMCH does not compensate for poor arithmetic in the upper half of
the exponent range, as is found on a Cray.

Parameters

SMALL

SMALL is DOUBLE PRECISION
On entry, the underflow threshold as computed by DLAMCH.
On exit, if LOG10(LARGE) is sufficiently large, the square
root of SMALL, otherwise unchanged.

LARGE

LARGE is DOUBLE PRECISION
On entry, the overflow threshold as computed by DLAMCH.
On exit, if LOG10(LARGE) is sufficiently large, the square



root of LARGE, otherwise unchanged.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine dlacpy (character UPLO, integer M, integer N, double precision, dimension(lda, *) A, integer LDA, double precision, dimension(ldb, *) B, integer LDB)

DLACPY copies all or part of one two-dimensional array to another.

Purpose:

DLACPY copies all or part of a two-dimensional matrix A to another matrix B.

Parameters

UPLO

UPLO is CHARACTER*1
Specifies the part of the matrix A to be copied to B.
= 'U': Upper triangular part
= 'L': Lower triangular part
Otherwise: All of the matrix A

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

A

A is DOUBLE PRECISION array, dimension (LDA,N)
The m by n matrix A. If UPLO = 'U', only the upper triangle or trapezoid is accessed; if UPLO = 'L', only the lower triangle or trapezoid is accessed.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

B

B is DOUBLE PRECISION array, dimension (LDB,N)
On exit, B = A in the locations specified by UPLO.

LDB

LDB is INTEGER
The leading dimension of the array B. LDB >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date



December 2016

subroutine dlae2 (double precision A, double precision B, double precision C, double precision RT1, double precision RT2)

DLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix.

Purpose:

DLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix
[A B]
[B C].
On return, RT1 is the eigenvalue of larger absolute value, and RT2
is the eigenvalue of smaller absolute value.

Parameters

A

A is DOUBLE PRECISION
The (1,1) element of the 2-by-2 matrix.

B

B is DOUBLE PRECISION
The (1,2) and (2,1) elements of the 2-by-2 matrix.

C

C is DOUBLE PRECISION
The (2,2) element of the 2-by-2 matrix.

RT1

RT1 is DOUBLE PRECISION
The eigenvalue of larger absolute value.

RT2

RT2 is DOUBLE PRECISION
The eigenvalue of smaller absolute value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

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December 2016

Further Details:

RT1 is accurate to a few ulps barring over/underflow.

RT2 may be inaccurate if there is massive cancellation in the
determinant A*C-B*B; higher precision or correctly rounded or
correctly truncated arithmetic would be needed to compute RT2
accurately in all cases.

Overflow is possible only if RT1 is within a factor of 5 of overflow.
Underflow is harmless if the input data is 0 or exceeds
underflow_threshold / macheps.

subroutine dlaebz (integer IJOB, integer NITMAX, integer N, integer MMAX, integer MINP, integer NBMIN, double precision ABSTOL, double precision RELTOL, double precision PIVMIN, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(*) E2, integer, dimension(*) NVAL, double precision, dimension(mmax, *) AB,



double precision, dimension(*) C, integer MOUT, integer, dimension(mmax, *) NAB, double precision, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)
DLAEBZ computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine sstebz.

Purpose:

DLAEBZ contains the iteration loops which compute and use the function N(w), which is the count of eigenvalues of a symmetric tridiagonal matrix T less than or equal to its argument w. It performs a choice of two types of loops:

IJOB=1, followed by
IJOB=2: It takes as input a list of intervals and returns a list of sufficiently small intervals whose union contains the same eigenvalues as the union of the original intervals.
The input intervals are (AB(j,1),AB(j,2)], j=1,...,MINP.
The output interval (AB(j,1),AB(j,2)] will contain eigenvalues NAB(j,1)+1,...,NAB(j,2), where 1 <= j <= MOUT.

IJOB=3: It performs a binary search in each input interval (AB(j,1),AB(j,2)] for a point w(j) such that N(w(j))=NVAL(j), and uses C(j) as the starting point of the search. If such a w(j) is found, then on output AB(j,1)=AB(j,2)=w. If no such w(j) is found, then on output (AB(j,1),AB(j,2)] will be a small interval containing the point where N(w) jumps through NVAL(j), unless that point lies outside the initial interval.

Note that the intervals are in all cases half-open intervals, i.e., of the form (a,b] , which includes b but not a .

To avoid underflow, the matrix should be scaled so that its largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value. To assure the most accurate computation of small eigenvalues, the matrix should be scaled to be not much smaller than that, either.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal Matrix", Report CS41, Computer Science Dept., Stanford University, July 21, 1966

Note: the arguments are, in general, *not* checked for unreasonable values.

Parameters

IJOB

IJOB is INTEGER
Specifies what is to be done:
= 1: Compute NAB for the initial intervals.
= 2: Perform bisection iteration to find eigenvalues of T.
= 3: Perform bisection iteration to invert N(w), i.e., to find a point which has a specified number of eigenvalues of T to its left.
Other values will cause DLAEBZ to return with INFO=-1.

NITMAX

NITMAX is INTEGER
The maximum number of "levels" of bisection to be



performed, i.e., an interval of width W will not be made
smaller than 2^(-NITMAX) * W. If not all intervals
have converged after NITMAX iterations, then INFO is set
to the number of non-converged intervals.

N

N is INTEGER
The dimension n of the tridiagonal matrix T. It must be at
least 1.

MMAX

MMAX is INTEGER
The maximum number of intervals. If more than MMAX intervals
are generated, then DLAEBZ will quit with INFO=MMAX+1.

MINP

MINP is INTEGER
The initial number of intervals. It may not be greater than
MMAX.

NBMIN

NBMIN is INTEGER
The smallest number of intervals that should be processed
using a vector loop. If zero, then only the scalar loop
will be used.

ABSTOL

ABSTOL is DOUBLE PRECISION
The minimum (absolute) width of an interval. When an
interval is narrower than ABSTOL, or than RELTOL times the
larger (in magnitude) endpoint, then it is considered to be
sufficiently small, i.e., converged. This must be at least
zero.

RELTOL

RELTOL is DOUBLE PRECISION
The minimum relative width of an interval. When an interval
is narrower than ABSTOL, or than RELTOL times the larger (in
magnitude) endpoint, then it is considered to be
sufficiently small, i.e., converged. Note: this should
always be at least radix*machine epsilon.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum absolute value of a "pivot" in the Sturm
sequence loop.
This must be at least max |e(j)**2|*safe_min and at
least safe_min, where safe_min is at least
the smallest number that can divide one without overflow.

D

D is DOUBLE PRECISION array, dimension (N)
The diagonal elements of the tridiagonal matrix T.

E

E is DOUBLE PRECISION array, dimension (N)
The offdiagonal elements of the tridiagonal matrix T in
positions 1 through N-1. E(N) is arbitrary.

E2

E2 is DOUBLE PRECISION array, dimension (N)



The squares of the offdiagonal elements of the tridiagonal matrix T. E2(N) is ignored.

NVAL

NVAL is INTEGER array, dimension (MINP)
If IJOB=1 or 2, not referenced.
If IJOB=3, the desired values of N(w). The elements of NVAL will be reordered to correspond with the intervals in AB. Thus, NVAL(j) on output will not, in general be the same as NVAL(j) on input, but it will correspond with the interval (AB(j,1),AB(j,2)] on output.

AB

AB is DOUBLE PRECISION array, dimension (MMAX,2)
The endpoints of the intervals. AB(j,1) is a(j), the left endpoint of the j-th interval, and AB(j,2) is b(j), the right endpoint of the j-th interval. The input intervals will, in general, be modified, split, and reordered by the calculation.

C

C is DOUBLE PRECISION array, dimension (MMAX)
If IJOB=1, ignored.
If IJOB=2, workspace.
If IJOB=3, then on input C(j) should be initialized to the first search point in the binary search.

MOUT

MOUT is INTEGER
If IJOB=1, the number of eigenvalues in the intervals.
If IJOB=2 or 3, the number of intervals output.
If IJOB=3, MOUT will equal MINP.

NAB

NAB is INTEGER array, dimension (MMAX,2)
If IJOB=1, then on output NAB(i,j) will be set to N(AB(i,j)).
If IJOB=2, then on input, NAB(i,j) should be set. It must satisfy the condition:
N(AB(i,1)) <= NAB(i,1) <= NAB(i,2) <= N(AB(i,2)),
which means that in interval i only eigenvalues NAB(i,1)+1,...,NAB(i,2) will be considered. Usually, NAB(i,j)=N(AB(i,j)), from a previous call to DLAEBZ with IJOB=1.
On output, NAB(i,j) will contain max(na(k),min(nb(k),N(AB(i,j)))), where k is the index of the input interval that the output interval (AB(j,1),AB(j,2)] came from, and na(k) and nb(k) are the the input values of NAB(k,1) and NAB(k,2).
If IJOB=3, then on output, NAB(i,j) contains N(AB(i,j)), unless N(w) > NVAL(i) for all search points w , in which case NAB(i,1) will not be modified, i.e., the output value will be the same as the input value (modulo reorderings -- see NVAL and AB), or unless N(w) < NVAL(i) for all search points w , in which case NAB(i,2) will not be modified. Normally, NAB should be set to some distinctive value(s) before DLAEBZ is called.

WORK

WORK is DOUBLE PRECISION array, dimension (MMAX)
Workspace.



IWORK

IWORK is INTEGER array, dimension (MMAX)
Workspace.

INFO

INFO is INTEGER
= 0: All intervals converged.
= 1--MMAX: The last INFO intervals did not converge.
= MMAX+1: More than MMAX intervals were generated.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

This routine is intended to be called only by other LAPACK routines, thus the interface is less user-friendly. It is intended for two purposes:

(a) finding eigenvalues. In this case, DLAEBZ should have one or more initial intervals set up in AB, and DLAEBZ should be called with IJOB=1. This sets up NAB, and also counts the eigenvalues. Intervals with no eigenvalues would usually be thrown out at this point. Also, if not all the eigenvalues in an interval i are desired, NAB(i,1) can be increased or NAB(i,2) decreased. For example, set NAB(i,1)=NAB(i,2)-1 to get the largest eigenvalue. DLAEBZ is then called with IJOB=2 and MMAX no smaller than the value of MOUT returned by the call with IJOB=1. After this (IJOB=2) call, eigenvalues NAB(i,1)+1 through NAB(i,2) are approximately AB(i,1) (or AB(i,2)) to the tolerance specified by ABSTOL and RELTOL.

(b) finding an interval (a',b'] containing eigenvalues w(f),...,w(l). In this case, start with a Gershgorin interval (a,b). Set up AB to contain 2 search intervals, both initially (a,b). One NVAL element should contain f-1 and the other should contain l , while C should contain a and b, resp. NAB(i,1) should be -1 and NAB(i,2) should be N+1, to flag an error if the desired interval does not lie in (a,b). DLAEBZ is then called with IJOB=3. On exit, if w(f-1) < w(f), then one of the intervals -- j -- will have AB(j,1)=AB(j,2) and NAB(j,1)=NAB(j,2)=f-1, while if, to the specified tolerance, w(f-k)=...=w(f+r), k > 0 and r >= 0, then the interval will have N(AB(j,1))=NAB(j,1)=f-k and N(AB(j,2))=NAB(j,2)=f+r. The cases w(l) < w(l+1) and w(l-r)=...=w(l+k) are handled similarly.

subroutine dlaev2 (double precision A, double precision B, double precision C, double precision RT1, double precision RT2, double precision CS1, double precision SN1)

DLAEV2 computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.

Purpose:

DLAEV2 computes the eigendecomposition of a 2-by-2 symmetric matrix



[A B]
[B C].

On return, RT1 is the eigenvalue of larger absolute value, RT2 is the eigenvalue of smaller absolute value, and (CS1,SN1) is the unit right eigenvector for RT1, giving the decomposition

[CS1 SN1] [A B] [CS1 -SN1] = [RT1 0]
[-SN1 CS1] [B C] [SN1 CS1] [0 RT2].

Parameters

A

A is DOUBLE PRECISION
The (1,1) element of the 2-by-2 matrix.

B

B is DOUBLE PRECISION
The (1,2) element and the conjugate of the (2,1) element of the 2-by-2 matrix.

C

C is DOUBLE PRECISION
The (2,2) element of the 2-by-2 matrix.

RT1

RT1 is DOUBLE PRECISION
The eigenvalue of larger absolute value.

RT2

RT2 is DOUBLE PRECISION
The eigenvalue of smaller absolute value.

CS1

CS1 is DOUBLE PRECISION

SN1

SN1 is DOUBLE PRECISION
The vector (CS1, SN1) is a unit right eigenvector for RT1.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

RT1 is accurate to a few ulps barring over/underflow.

RT2 may be inaccurate if there is massive cancellation in the determinant A*C-B*B; higher precision or correctly rounded or correctly truncated arithmetic would be needed to compute RT2 accurately in all cases.

CS1 and SN1 are accurate to a few ulps barring over/underflow.

Overflow is possible only if RT1 is within a factor of 5 of overflow. Underflow is harmless if the input data is 0 or exceeds



underflow_threshold / macheps.

subroutine dlagts (integer JOB, integer N, double precision, dimension(*) A, double precision, dimension(*) B, double precision, dimension(*) C, double precision, dimension(*) D, integer, dimension(*) IN, double precision, dimension(*) Y, double precision TOL, integer INFO)

DLAGTS solves the system of equations (T-λI)x = y or (T-λI)Tx = y,where T is a general tridiagonal matrix and λ a scalar, using the LU factorization computed by slagtf.

Purpose:

DLAGTS may be used to solve one of the systems of equations

(T - lambda*I)*x = y or (T - lambda*I)**T*x = y,

where T is an n by n tridiagonal matrix, for x, following the factorization of (T - lambda*I) as

(T - lambda*I) = P*L*U ,

by routine DLAGTF. The choice of equation to be solved is controlled by the argument JOB, and in each case there is an option to perturb zero or very small diagonal elements of U, this option being intended for use in applications such as inverse iteration.

Parameters

JOB

JOB is INTEGER
Specifies the job to be performed by DLAGTS as follows:
= 1: The equations (T - lambda*I)x = y are to be solved, but diagonal elements of U are not to be perturbed.
= -1: The equations (T - lambda*I)x = y are to be solved and, if overflow would otherwise occur, the diagonal elements of U are to be perturbed. See argument TOL below.
= 2: The equations (T - lambda*I)**Tx = y are to be solved, but diagonal elements of U are not to be perturbed.
= -2: The equations (T - lambda*I)**Tx = y are to be solved and, if overflow would otherwise occur, the diagonal elements of U are to be perturbed. See argument TOL below.

N

N is INTEGER
The order of the matrix T.

A

A is DOUBLE PRECISION array, dimension (N)
On entry, A must contain the diagonal elements of U as returned from DLAGTF.

B

B is DOUBLE PRECISION array, dimension (N-1)
On entry, B must contain the first super-diagonal elements of U as returned from DLAGTF.

C

C is DOUBLE PRECISION array, dimension (N-1)
On entry, C must contain the sub-diagonal elements of L as returned from DLAGTF.

D



D is DOUBLE PRECISION array, dimension (N-2)
On entry, D must contain the second super-diagonal elements of U as returned from DLAGTF.

IN

IN is INTEGER array, dimension (N)
On entry, IN must contain details of the matrix P as returned from DLAGTF.

Y

Y is DOUBLE PRECISION array, dimension (N)
On entry, the right hand side vector y.
On exit, Y is overwritten by the solution vector x.

TOL

TOL is DOUBLE PRECISION
On entry, with JOB < 0, TOL should be the minimum perturbation to be made to very small diagonal elements of U. TOL should normally be chosen as about eps*norm(U), where eps is the relative machine precision, but if TOL is supplied as non-positive, then it is reset to eps*max(abs(u(i,j))). If JOB > 0 then TOL is not referenced.

On exit, TOL is changed as described above, only if TOL is non-positive on entry. Otherwise TOL is unchanged.

INFO

INFO is INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: overflow would occur when computing the INFO(th) element of the solution vector x. This can only occur when JOB is supplied as positive and either means that a diagonal element of U is very small, or that the elements of the right-hand side vector y are very large.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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logical function dlaisnan (double precision, intent(in) DIN1, double precision, intent(in) DIN2)

DLAISNAN tests input for NaN by comparing two arguments for inequality.

Purpose:

This routine is not for general use. It exists solely to avoid over-optimization in DISNAN.

DLAISNAN checks for NaNs by comparing its two arguments for inequality. NaN is the only floating-point value where NaN != NaN returns .TRUE. To check for NaNs, pass the same variable as both arguments.

A compiler must assume that the two arguments are



not the same variable, and the test will not be optimized away. Interprocedural or whole-program optimization may delete this test. The ISNAN functions will be replaced by the correct Fortran 03 intrinsic once the intrinsic is widely available.

Parameters

DIN1

DIN1 is DOUBLE PRECISION

DIN2

DIN2 is DOUBLE PRECISION
Two numbers to compare for inequality.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

integer function dlaneg (integer N, double precision, dimension(*) D, double precision, dimension(*) LLD, double precision SIGMA, double precision PIVMIN, integer R)

DLANEG computes the Sturm count.

Purpose:

DLANEG computes the Sturm count, the number of negative pivots encountered while factoring tridiagonal T - sigma I = L D L^T. This implementation works directly on the factors without forming the tridiagonal matrix T. The Sturm count is also the number of eigenvalues of T less than sigma.

This routine is called from DLARRB.

The current routine does not use the PIVMIN parameter but rather requires IEEE-754 propagation of Infinities and NaNs. This routine also has no input range restrictions but does require default exception handling such that x/0 produces Inf when x is non-zero, and Inf/Inf produces NaN. For more information, see:

Marques, Riedy, and Voemel, "Benefits of IEEE-754 Features in Modern Symmetric Tridiagonal Eigensolvers," SIAM Journal on Scientific Computing, v28, n5, 2006. DOI 10.1137/050641624 (Tech report version in LAWN 172 with the same title.)

Parameters

N

N is INTEGER
The order of the matrix.

D

D is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of the diagonal matrix D.

LLD

LLD is DOUBLE PRECISION array, dimension (N-1)
The (N-1) elements L(i)*L(i)*D(i).

SIGMA



SIGMA is DOUBLE PRECISION
Shift amount in T - sigma I = L D L^T.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot in the Sturm sequence. May be used when zero pivots are encountered on non-IEEE-754 architectures.

R

R is INTEGER
The twist index for the twisted factorization that is used for the negcount.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA
Jason Riedy, University of California, Berkeley, USA

double precision function dlanst (character NORM, integer N, double precision, dimension(*) D, double precision, dimension(*) E)

DLANST returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix.

Purpose:

DLANST returns the value of the one norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix A.

Returns

DLANST

DLANST = (max(abs(A(i,j))), NORM = 'M' or 'm'
(
(norm1(A), NORM = '1', 'O' or 'o'
(
(normI(A), NORM = 'I' or 'i'
(
(normF(A), NORM = 'F', 'f', 'E' or 'e'

where norm1 denotes the one norm of a matrix (maximum column sum), normI denotes the infinity norm of a matrix (maximum row sum) and normF denotes the Frobenius norm of a matrix (square root of sum of squares). Note that max(abs(A(i,j))) is not a consistent matrix norm.

Parameters

NORM

NORM is CHARACTER*1
Specifies the value to be returned in DLANST as described above.

N



N is INTEGER
The order of the matrix A. N >= 0. When N = 0, DLANST is set to zero.

D

D is DOUBLE PRECISION array, dimension (N)
The diagonal elements of A.

E

E is DOUBLE PRECISION array, dimension (N-1)
The (n-1) sub-diagonal or super-diagonal elements of A.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

double precision function dlapy2 (double precision X, double precision Y)

DLAPY2 returns sqrt(x2+y2).

Purpose:

DLAPY2 returns sqrt(x**2+y**2), taking care not to cause unnecessary overflow.

Parameters

X

X is DOUBLE PRECISION

Y

Y is DOUBLE PRECISION
X and Y specify the values x and y.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

double precision function dlapy3 (double precision X, double precision Y, double precision Z)

DLAPY3 returns sqrt(x2+y2+z2).

Purpose:

DLAPY3 returns sqrt(x**2+y**2+z**2), taking care not to cause unnecessary overflow.

Parameters

X

X is DOUBLE PRECISION

Y

Y is DOUBLE PRECISION

Z



Z is DOUBLE PRECISION
X, Y and Z specify the values x, y and z.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine dlarnv (integer IDIST, integer, dimension(4) ISEED, integer N, double precision, dimension(*) X)

DLARNV returns a vector of random numbers from a uniform or normal distribution.

Purpose:

DLARNV returns a vector of n random real numbers from a uniform or normal distribution.

Parameters

IDIST

IDIST is INTEGER
Specifies the distribution of the random numbers:
= 1: uniform (0,1)
= 2: uniform (-1,1)
= 3: normal (0,1)

ISEED

ISEED is INTEGER array, dimension (4)
On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and ISEED(4) must be odd.
On exit, the seed is updated.

N

N is INTEGER
The number of random numbers to be generated.

X

X is DOUBLE PRECISION array, dimension (N)
The generated random numbers.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

This routine calls the auxiliary routine DLARUV to generate random real numbers from a uniform (0,1) distribution, in batches of up to 128 using vectorisable code. The Box-Muller method is used to transform numbers from a uniform to a normal distribution.



subroutine dlarra (integer N, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(*) E2, double precision SPLTOL, double precision TNRM, integer NSPLIT, integer, dimension(*) ISPLIT, integer INFO)

DLARRA computes the splitting points with the specified threshold.

Purpose:

Compute the splitting points with threshold SPLTOL.
DLARRA sets any "small" off-diagonal elements to zero.

Parameters

N

N is INTEGER
The order of the matrix. N > 0.

D

D is DOUBLE PRECISION array, dimension (N)
On entry, the N diagonal elements of the tridiagonal
matrix T.

E

E is DOUBLE PRECISION array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal
elements of the tridiagonal matrix T; E(N) need not be set.
On exit, the entries E(ISPLIT(I)), 1 <= I <= NSPLIT,
are set to zero, the other entries of E are untouched.

E2

E2 is DOUBLE PRECISION array, dimension (N)
On entry, the first (N-1) entries contain the SQUARES of the
subdiagonal elements of the tridiagonal matrix T;
E2(N) need not be set.
On exit, the entries E2(ISPLIT(I)),
1 <= I <= NSPLIT, have been set to zero

SPLTOL

SPLTOL is DOUBLE PRECISION
The threshold for splitting. Two criteria can be used:
SPLTOL<0 : criterion based on absolute off-diagonal value
SPLTOL>0 : criterion that preserves relative accuracy

TNRM

TNRM is DOUBLE PRECISION
The norm of the matrix.

NSPLIT

NSPLIT is INTEGER
The number of blocks T splits into. 1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into blocks.
The first block consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.

INFO

INFO is INTEGER
= 0: successful exit



Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrb (integer N, double precision, dimension(*) D, double precision, dimension(*) LLD, integer IFIRST, integer ILAST, double precision RTOL1, double precision RTOL2, integer OFFSET, double precision, dimension(*) W, double precision, dimension(*) WGAP, double precision, dimension(*) WERR, double precision, dimension(*) WORK, integer, dimension(*) IWORK, double precision PIVMIN, double precision SPDIAM, integer TWIST, integer INFO)

DLARRB provides limited bisection to locate eigenvalues for more accuracy.

Purpose:

Given the relatively robust representation(RRR) L D L^T, DLARRB
does "limited" bisection to refine the eigenvalues of L D L^T,
W(IFIRST-OFFSET) through W(ILAST-OFFSET), to more accuracy. Initial
guesses for these eigenvalues are input in W, the corresponding estimate
of the error in these guesses and their gaps are input in WERR
and WGAP, respectively. During bisection, intervals
[left, right] are maintained by storing their mid-points and
semi-widths in the arrays W and WERR respectively.

Parameters

N

N is INTEGER
The order of the matrix.

D

D is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of the diagonal matrix D.

LLD

LLD is DOUBLE PRECISION array, dimension (N-1)
The (N-1) elements L(i)*L(i)*D(i).

IFIRST

IFIRST is INTEGER
The index of the first eigenvalue to be computed.

ILAST

ILAST is INTEGER
The index of the last eigenvalue to be computed.

RTOL1

RTOL1 is DOUBLE PRECISION

RTOL2

RTOL2 is DOUBLE PRECISION
Tolerance for the convergence of the bisection intervals.



An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < MAX(RTOL1*GAP, RTOL2*MAX(|LEFT|,|RIGHT|))
where GAP is the (estimated) distance to the nearest
eigenvalue.

OFFSET

OFFSET is INTEGER
Offset for the arrays W, WGAP and WERR, i.e., the IFIRST-OFFSET
through ILAST-OFFSET elements of these arrays are to be used.

W

W is DOUBLE PRECISION array, dimension (N)
On input, W(IFIRST-OFFSET) through W(ILAST-OFFSET) are
estimates of the eigenvalues of L D L^T indexed IFIRST through
ILAST.
On output, these estimates are refined.

WGAP

WGAP is DOUBLE PRECISION array, dimension (N-1)
On input, the (estimated) gaps between consecutive
eigenvalues of L D L^T, i.e., WGAP(I-OFFSET) is the gap between
eigenvalues I and I+1. Note that if IFIRST = ILAST
then WGAP(IFIRST-OFFSET) must be set to ZERO.
On output, these gaps are refined.

WERR

WERR is DOUBLE PRECISION array, dimension (N)
On input, WERR(IFIRST-OFFSET) through WERR(ILAST-OFFSET) are
the errors in the estimates of the corresponding elements in W.
On output, these errors are refined.

WORK

WORK is DOUBLE PRECISION array, dimension (2*N)
Workspace.

IWORK

IWORK is INTEGER array, dimension (2*N)
Workspace.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot in the Sturm sequence.

SPDIAM

SPDIAM is DOUBLE PRECISION
The spectral diameter of the matrix.

TWIST

TWIST is INTEGER
The twist index for the twisted factorization that is used
for the negcount.
TWIST = N: Compute negcount from L D L^T - LAMBDA I = L+ D+ L+^T
TWIST = 1: Compute negcount from L D L^T - LAMBDA I = U- D- U-^T
TWIST = R: Compute negcount from L D L^T - LAMBDA I = N(r) D(r) N(r)

INFO

INFO is INTEGER
Error flag.

Author

Univ. of Tennessee



Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date
June 2017

Contributors:
Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrc (character JOBT, integer N, double precision VL, double precision VU, double precision, dimension(*) D, double precision, dimension(*) E, double precision PIVMIN, integer EIGCNT, integer LCNT, integer RCNT, integer INFO)
DLARRC computes the number of eigenvalues of the symmetric tridiagonal matrix.

Purpose:

Find the number of eigenvalues of the symmetric tridiagonal matrix T
that are in the interval (VL,VU] if JOBT = 'T', and of L D L^T
if JOBT = 'L'.

Parameters

JOBT

JOBT is CHARACTER*1
= 'T': Compute Sturm count for matrix T.
= 'L': Compute Sturm count for matrix L D L^T.

N

N is INTEGER
The order of the matrix. N > 0.

VL

VL is DOUBLE PRECISION
The lower bound for the eigenvalues.

VU

VU is DOUBLE PRECISION
The upper bound for the eigenvalues.

D

D is DOUBLE PRECISION array, dimension (N)
JOBT = 'T': The N diagonal elements of the tridiagonal matrix T.
JOBT = 'L': The N diagonal elements of the diagonal matrix D.

E

E is DOUBLE PRECISION array, dimension (N)
JOBT = 'T': The N-1 offdiagonal elements of the matrix T.
JOBT = 'L': The N-1 offdiagonal elements of the matrix L.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot in the Sturm sequence for T.

EIGCNT

EIGCNT is INTEGER
The number of eigenvalues of the symmetric tridiagonal matrix T
that are in the interval (VL,VU]



LCNT

LCNT is INTEGER

RCNT

RCNT is INTEGER
The left and right negcounts of the interval.

INFO

INFO is INTEGER

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrd (character RANGE, character ORDER, integer N, double precision VL, double precision VU, integer IL, integer IU, double precision, dimension(*) GERS, double precision RELTOL, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(*) E2, double precision PIVMIN, integer NSPLIT, integer, dimension(*) ISPLIT, integer M, double precision, dimension(*) W, double precision, dimension(*) WERR, double precision WL, double precision WU, integer, dimension(*) IBLOCK, integer, dimension(*) INDEXW, double precision, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

DLARRD computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

Purpose:

DLARRD computes the eigenvalues of a symmetric tridiagonal
matrix T to suitable accuracy. This is an auxiliary code to be
called from DSTEMR.
The user may ask for all eigenvalues, all eigenvalues
in the half-open interval (VL, VU], or the IL-th through IU-th
eigenvalues.

To avoid overflow, the matrix must be scaled so that its
largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal
Matrix", Report CS41, Computer Science Dept., Stanford
University, July 21, 1966.

Parameters

RANGE

RANGE is CHARACTER*1
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval
(VL, VU] will be found.
= 'I': ("Index") the IL-th through IU-th eigenvalues (of the



entire matrix) will be found.

ORDER

ORDER is CHARACTER*1
= 'B': ("By Block") the eigenvalues will be grouped by
split-off block (see IBLOCK, ISPLIT) and
ordered from smallest to largest within
the block.
= 'E': ("Entire matrix")
the eigenvalues for the entire matrix
will be ordered from smallest to
largest.

N

N is INTEGER
The order of the tridiagonal matrix T. N >= 0.

VL

VL is DOUBLE PRECISION
If RANGE='V', the lower bound of the interval to
be searched for eigenvalues. Eigenvalues less than or equal
to VL, or greater than VU, will not be returned. VL < VU.
Not referenced if RANGE = 'A' or 'I'.

VU

VU is DOUBLE PRECISION
If RANGE='V', the upper bound of the interval to
be searched for eigenvalues. Eigenvalues less than or equal
to VL, or greater than VU, will not be returned. VL < VU.
Not referenced if RANGE = 'A' or 'I'.

IL

IL is INTEGER
If RANGE='I', the index of the
smallest eigenvalue to be returned.
1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0.
Not referenced if RANGE = 'A' or 'V'.

IU

IU is INTEGER
If RANGE='I', the index of the
largest eigenvalue to be returned.
1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0.
Not referenced if RANGE = 'A' or 'V'.

GERS

GERS is DOUBLE PRECISION array, dimension (2*N)
The N Gerschgorin intervals (the i-th Gerschgorin interval
is (GERS(2*i-1), GERS(2*i)).

RELTOL

RELTOL is DOUBLE PRECISION
The minimum relative width of an interval. When an interval
is narrower than RELTOL times the larger (in
magnitude) endpoint, then it is considered to be
sufficiently small, i.e., converged. Note: this should
always be at least radix*machine epsilon.

D

D is DOUBLE PRECISION array, dimension (N)



The n diagonal elements of the tridiagonal matrix T.

E

E is DOUBLE PRECISION array, dimension (N-1)
The (n-1) off-diagonal elements of the tridiagonal matrix T.

E2

E2 is DOUBLE PRECISION array, dimension (N-1)
The (n-1) squared off-diagonal elements of the tridiagonal matrix T.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot allowed in the Sturm sequence for T.

NSPLIT

NSPLIT is INTEGER
The number of diagonal blocks in the matrix T.
1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into submatrices.
The first submatrix consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.
(Only the first NSPLIT elements will actually be used, but
since the user cannot know a priori what value NSPLIT will
have, N words must be reserved for ISPLIT.)

M

M is INTEGER
The actual number of eigenvalues found. 0 <= M <= N.
(See also the description of INFO=2,3.)

W

W is DOUBLE PRECISION array, dimension (N)
On exit, the first M elements of W will contain the
eigenvalue approximations. DLARRD computes an interval
I_j = (a_j, b_j] that includes eigenvalue j. The eigenvalue
approximation is given as the interval midpoint
W(j)= (a_j + b_j)/2. The corresponding error is bounded by
WERR(j) = abs(a_j - b_j)/2

WERR

WERR is DOUBLE PRECISION array, dimension (N)
The error bound on the corresponding eigenvalue approximation
in W.

WL

WL is DOUBLE PRECISION

WU

WU is DOUBLE PRECISION
The interval (WL, WU] contains all the wanted eigenvalues.
If RANGE='V', then WL=VL and WU=VU.
If RANGE='A', then WL and WU are the global Gerschgorin bounds
on the spectrum.
If RANGE='I', then WL and WU are computed by DLAEBZ from the
index range specified.



IBLOCK

IBLOCK is INTEGER array, dimension (N)
At each row/column j where E(j) is zero or small, the matrix T is considered to split into a block diagonal matrix. On exit, if INFO = 0, IBLOCK(i) specifies to which block (from 1 to the number of blocks) the eigenvalue W(i) belongs. (DLARRD may use the remaining N-M elements as workspace.)

INDEXW

INDEXW is INTEGER array, dimension (N)
The indices of the eigenvalues within each block (submatrix); for example, INDEXW(i)= j and IBLOCK(i)=k imply that the i-th eigenvalue W(i) is the j-th eigenvalue in block k.

WORK

WORK is DOUBLE PRECISION array, dimension (4*N)

IWORK

IWORK is INTEGER array, dimension (3*N)

INFO

INFO is INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: some or all of the eigenvalues failed to converge or were not computed:
=1 or 3: Bisection failed to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. This is generally caused by unexpectedly inaccurate arithmetic.
=2 or 3: RANGE='I' only: Not all of the eigenvalues IL:IU were found.
Effect: M < IU+1-IL
Cause: non-monotonic arithmetic, causing the Sturm sequence to be non-monotonic.
Cure: recalculate, using RANGE='A', and pick out eigenvalues IL:IU. In some cases, increasing the PARAMETER "FUDGE" may make things work.
= 4: RANGE='I', and the Gershgorin interval initially used was too small. No eigenvalues were computed.
Probable cause: your machine has sloppy floating-point arithmetic.
Cure: Increase the PARAMETER "FUDGE", recompile, and try again.

Internal Parameters:

FUDGE DOUBLE PRECISION, default = 2
A "fudge factor" to widen the Gershgorin intervals. Ideally, a value of 1 should work, but on machines with sloppy arithmetic, this needs to be larger. The default for publicly released versions should be large enough to handle the worst machine around. Note that this has no effect



on accuracy of the solution.

Contributors:

W. Kahan, University of California, Berkeley, USA
Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine dlarre (character RANGE, integer N, double precision VL, double precision VU, integer IL, integer IU, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(*) E2, double precision RTOL1, double precision RTOL2, double precision SPLTOL, integer NSPLIT, integer, dimension(*) ISPLIT, integer M, double precision, dimension(*) W, double precision, dimension(*) WERR, double precision, dimension(*) WGAP, integer, dimension(*) IBLOCK, integer, dimension(*) INDEXW, double precision, dimension(*) GERS, double precision PIVMIN, double precision, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

DLARRE given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block Ti, finds base representations and eigenvalues.

Purpose:

To find the desired eigenvalues of a given real symmetric
tridiagonal matrix T, DLARRE sets any "small" off-diagonal
elements to zero, and for each unreduced block T_i, it finds
(a) a suitable shift at one end of the block's spectrum,
(b) the base representation, T_i - sigma_i I = L_i D_i L_iˆT, and
(c) eigenvalues of each L_i D_i L_iˆT.
The representations and eigenvalues found are then used by
DSTEMR to compute the eigenvectors of T.
The accuracy varies depending on whether bisection is used to
find a few eigenvalues or the dqds algorithm (subroutine DLASQ2) to
conpute all and then discard any unwanted one.
As an added benefit, DLARRE also outputs the n
Gerschgorin intervals for the matrices L_i D_i L_iˆT.

Parameters

RANGE

RANGE is CHARACTER*1
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval
(VL, VU] will be found.
= 'I': ("Index") the IL-th through IU-th eigenvalues (of the
entire matrix) will be found.

N

N is INTEGER
The order of the matrix. N > 0.

VL

VL is DOUBLE PRECISION



If RANGE='V', the lower bound for the eigenvalues.
Eigenvalues less than or equal to VL, or greater than VU,
will not be returned. VL < VU.
If RANGE='I' or ='A', DLARRE computes bounds on the desired
part of the spectrum.

VU

VU is DOUBLE PRECISION
If RANGE='V', the upper bound for the eigenvalues.
Eigenvalues less than or equal to VL, or greater than VU,
will not be returned. VL < VU.
If RANGE='I' or ='A', DLARRE computes bounds on the desired
part of the spectrum.

IL

IL is INTEGER
If RANGE='I', the index of the
smallest eigenvalue to be returned.
1 <= IL <= IU <= N.

IU

IU is INTEGER
If RANGE='I', the index of the
largest eigenvalue to be returned.
1 <= IL <= IU <= N.

D

D is DOUBLE PRECISION array, dimension (N)
On entry, the N diagonal elements of the tridiagonal
matrix T.
On exit, the N diagonal elements of the diagonal
matrices D_i.

E

E is DOUBLE PRECISION array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal
elements of the tridiagonal matrix T; E(N) need not be set.
On exit, E contains the subdiagonal elements of the unit
bidiagonal matrices L_i. The entries E(ISPLIT(I)),
1 <= I <= NSPLIT, contain the base points sigma_i on output.

E2

E2 is DOUBLE PRECISION array, dimension (N)
On entry, the first (N-1) entries contain the SQUARES of the
subdiagonal elements of the tridiagonal matrix T;
E2(N) need not be set.
On exit, the entries E2(ISPLIT(I)),
1 <= I <= NSPLIT, have been set to zero

RTOL1

RTOL1 is DOUBLE PRECISION

RTOL2

RTOL2 is DOUBLE PRECISION
Parameters for bisection.
An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < MAX(RTOL1*GAP, RTOL2*MAX(|LEFT|,|RIGHT|))

SPLTOL

SPLTOL is DOUBLE PRECISION



The threshold for splitting.

NSPLIT

NSPLIT is INTEGER
The number of blocks T splits into. 1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into blocks.
The first block consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.

M

M is INTEGER
The total number of eigenvalues (of all L_i D_i L_iˆT)
found.

W

W is DOUBLE PRECISION array, dimension (N)
The first M elements contain the eigenvalues. The
eigenvalues of each of the blocks, L_i D_i L_iˆT, are
sorted in ascending order (DLARRE may use the
remaining N-M elements as workspace).

WERR

WERR is DOUBLE PRECISION array, dimension (N)
The error bound on the corresponding eigenvalue in W.

WGAP

WGAP is DOUBLE PRECISION array, dimension (N)
The separation from the right neighbor eigenvalue in W.
The gap is only with respect to the eigenvalues of the same block
as each block has its own representation tree.
Exception: at the right end of a block we store the left gap

IBLOCK

IBLOCK is INTEGER array, dimension (N)
The indices of the blocks (submatrices) associated with the
corresponding eigenvalues in W; IBLOCK(i)=1 if eigenvalue
W(i) belongs to the first block from the top, =2 if W(i)
belongs to the second block, etc.

INDEXW

INDEXW is INTEGER array, dimension (N)
The indices of the eigenvalues within each block (submatrix);
for example, INDEXW(i)= 10 and IBLOCK(i)=2 imply that the
i-th eigenvalue W(i) is the 10-th eigenvalue in block 2

GERS

GERS is DOUBLE PRECISION array, dimension (2*N)
The N Gerschgorin intervals (the i-th Gerschgorin interval
is (GERS(2*i-1), GERS(2*i)).

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot in the Sturm sequence for T.

WORK

WORK is DOUBLE PRECISION array, dimension (6*N)



Workspace.

IWORK

IWORK is INTEGER array, dimension (5*N)
Workspace.

INFO

INFO is INTEGER
= 0: successful exit
> 0: A problem occurred in DLARRE.
< 0: One of the called subroutines signaled an internal problem.
Needs inspection of the corresponding parameter IINFO
for further information.

=-1: Problem in DLARRD.
= 2: No base representation could be found in MAXTRY iterations.
Increasing MAXTRY and recompilation might be a remedy.
=-3: Problem in DLARRB when computing the refined root
representation for DLASQ2.
=-4: Problem in DLARRB when preforming bisection on the
desired part of the spectrum.
=-5: Problem in DLASQ2.
=-6: Problem in DLASQ2.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Further Details:

The base representations are required to suffer very little
element growth and consequently define all their eigenvalues to
high relative accuracy.

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrf (integer N, double precision, dimension(*) D, double precision, dimension(*) L, double precision, dimension(*) LD, integer CLSTRT, integer CLEND, double precision, dimension(*) W, double precision, dimension(*) WGAP, double precision, dimension(*) WERR, double precision SPDIAM, double precision CLGAPL, double precision CLGAPR, double precision PIVMIN, double precision SIGMA, double precision, dimension(*) DPLUS, double precision, dimension(*) LPLUS, double precision, dimension(*) WORK, integer INFO)

DLARRF finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

Purpose:

Given the initial representation L D L^T and its cluster of close
eigenvalues (in a relative measure), W(CLSTRT), W(CLSTRT+1), ...
W(CLEND), DLARRF finds a new relatively robust representation
L D L^T - SIGMA I = L(+) D(+) L(+)^T such that at least one of the



eigenvalues of L(+) D(+) L(+)ˆT is relatively isolated.

Parameters

N

N is INTEGER
The order of the matrix (subblock, if the matrix split).

D

D is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of the diagonal matrix D.

L

L is DOUBLE PRECISION array, dimension (N-1)
The (N-1) subdiagonal elements of the unit bidiagonal
matrix L.

LD

LD is DOUBLE PRECISION array, dimension (N-1)
The (N-1) elements L(i)*D(i).

CLSTRT

CLSTRT is INTEGER
The index of the first eigenvalue in the cluster.

CLEND

CLEND is INTEGER
The index of the last eigenvalue in the cluster.

W

W is DOUBLE PRECISION array, dimension
dimension is >= (CLEND-CLSTRT+1)
The eigenvalue APPROXIMATIONS of L D Lˆ T in ascending order.
W(CLSTRT) through W(CLEND) form the cluster of relatively
close eigenalues.

WGAP

WGAP is DOUBLE PRECISION array, dimension
dimension is >= (CLEND-CLSTRT+1)
The separation from the right neighbor eigenvalue in W.

WERR

WERR is DOUBLE PRECISION array, dimension
dimension is >= (CLEND-CLSTRT+1)
WERR contain the semiwidth of the uncertainty
interval of the corresponding eigenvalue APPROXIMATION in W

SPDIAM

SPDIAM is DOUBLE PRECISION
estimate of the spectral diameter obtained from the
Gerschgorin intervals

CLGAPL

CLGAPL is DOUBLE PRECISION

CLGAPR

CLGAPR is DOUBLE PRECISION
absolute gap on each end of the cluster.
Set by the calling routine to protect against shifts too close
to eigenvalues outside the cluster.

PIVMIN



PIVMIN is DOUBLE PRECISION
The minimum pivot allowed in the Sturm sequence.

SIGMA

SIGMA is DOUBLE PRECISION
The shift used to form L(+) D(+) L(+)ˆT.

DPLUS

DPLUS is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of the diagonal matrix D(+).

LPLUS

LPLUS is DOUBLE PRECISION array, dimension (N-1)
The first (N-1) elements of LPLUS contain the subdiagonal elements of the unit bidiagonal matrix L(+).

WORK

WORK is DOUBLE PRECISION array, dimension (2*N)
Workspace.

INFO

INFO is INTEGER
Signals processing OK (=0) or failure (=1)

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrj (integer N, double precision, dimension(*) D, double precision, dimension(*) E2, integer IFIRST, integer ILAST, double precision RTOL, integer OFFSET, double precision, dimension(*) W, double precision, dimension(*) WERR, double precision, dimension(*) WORK, integer, dimension(*) IWORK, double precision PIVMIN, double precision SPDIAM, integer INFO)

DLARRJ performs refinement of the initial estimates of the eigenvalues of the matrix T.

Purpose:

Given the initial eigenvalue approximations of T, DLARRJ
does bisection to refine the eigenvalues of T,
W(IFIRST-OFFSET) through W(ILAST-OFFSET), to more accuracy. Initial
guesses for these eigenvalues are input in W, the corresponding estimate
of the error in these guesses in WERR. During bisection, intervals
[left, right] are maintained by storing their mid-points and
semi-widths in the arrays W and WERR respectively.

Parameters

N

N is INTEGER
The order of the matrix.



D

D is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of T.

E2

E2 is DOUBLE PRECISION array, dimension (N-1)
The Squares of the (N-1) subdiagonal elements of T.

IFIRST

IFIRST is INTEGER
The index of the first eigenvalue to be computed.

ILAST

ILAST is INTEGER
The index of the last eigenvalue to be computed.

RTOL

RTOL is DOUBLE PRECISION
Tolerance for the convergence of the bisection intervals.
An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < RTOL*MAX(|LEFT|,|RIGHT|).

OFFSET

OFFSET is INTEGER
Offset for the arrays W and WERR, i.e., the IFIRST-OFFSET
through ILAST-OFFSET elements of these arrays are to be used.

W

W is DOUBLE PRECISION array, dimension (N)
On input, W(IFIRST-OFFSET) through W(ILAST-OFFSET) are
estimates of the eigenvalues of L D L^T indexed IFIRST through
ILAST.
On output, these estimates are refined.

WERR

WERR is DOUBLE PRECISION array, dimension (N)
On input, WERR(IFIRST-OFFSET) through WERR(ILAST-OFFSET) are
the errors in the estimates of the corresponding elements in W.
On output, these errors are refined.

WORK

WORK is DOUBLE PRECISION array, dimension (2*N)
Workspace.

IWORK

IWORK is INTEGER array, dimension (2*N)
Workspace.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot in the Sturm sequence for T.

SPDIAM

SPDIAM is DOUBLE PRECISION
The spectral diameter of T.

INFO

INFO is INTEGER
Error flag.

Author



Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlarrk (integer N, integer IW, double precision GL, double precision GU, double precision, dimension(*) D, double precision, dimension(*) E2, double precision PIVMIN, double precision RELTOL, double precision W, double precision WERR, integer INFO)

DLARRK computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.

Purpose:

DLARRK computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy. This is an auxiliary code to be called from DSTEMR.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal Matrix", Report CS41, Computer Science Dept., Stanford University, July 21, 1966.

Parameters

N

N is INTEGER
The order of the tridiagonal matrix T. N >= 0.

IW

IW is INTEGER
The index of the eigenvalues to be returned.

GL

GL is DOUBLE PRECISION

GU

GU is DOUBLE PRECISION
An upper and a lower bound on the eigenvalue.

D

D is DOUBLE PRECISION array, dimension (N)
The n diagonal elements of the tridiagonal matrix T.

E2

E2 is DOUBLE PRECISION array, dimension (N-1)
The (n-1) squared off-diagonal elements of the tridiagonal matrix T.

PIVMIN

PIVMIN is DOUBLE PRECISION
The minimum pivot allowed in the Sturm sequence for T.



RELTOL

RELTOL is DOUBLE PRECISION
The minimum relative width of an interval. When an interval is narrower than RELTOL times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. Note: this should always be at least radix*machine epsilon.

W

W is DOUBLE PRECISION

WERR

WERR is DOUBLE PRECISION
The error bound on the corresponding eigenvalue approximation in W.

INFO

INFO is INTEGER
= 0: Eigenvalue converged
= -1: Eigenvalue did NOT converge

Internal Parameters:

FUDGE DOUBLE PRECISION, default = 2
A "fudge factor" to widen the Gershgorin intervals.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

subroutine dlarrr (integer N, double precision, dimension(*) D, double precision, dimension(*) E, integer INFO)

DLARRR performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

Purpose:

Perform tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

Parameters

N

N is INTEGER
The order of the matrix. N > 0.

D

D is DOUBLE PRECISION array, dimension (N)
The N diagonal elements of the tridiagonal matrix T.

E

E is DOUBLE PRECISION array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal elements of the tridiagonal matrix T; E(N) is set to ZERO.

INFO



INFO is INTEGER
INFO = 0(default) : the matrix warrants computations preserving
relative accuracy.
INFO = 1 : the matrix warrants computations guaranteeing
only absolute accuracy.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine dlartg (double precision F, double precision G, double precision CS, double precision SN, double precision R)

DLARTG generates a plane rotation with real cosine and real sine.

Purpose:

DLARTG generate a plane rotation so that

[CS SN] . [F] = [R] where CS**2 + SN**2 = 1.
[-SN CS] [G] [0]

This is a slower, more accurate version of the BLAS1 routine DROTG,
with the following other differences:
F and G are unchanged on return.
If G=0, then CS=1 and SN=0.
If F=0 and (G .ne. 0), then CS=0 and SN=1 without doing any
floating point operations (saves work in DBDSQR when
there are zeros on the diagonal).

If F exceeds G in magnitude, CS will be positive.

Parameters

F

F is DOUBLE PRECISION
The first component of vector to be rotated.

G

G is DOUBLE PRECISION
The second component of vector to be rotated.

CS

CS is DOUBLE PRECISION
The cosine of the rotation.

SN

SN is DOUBLE PRECISION
The sine of the rotation.

R



R is DOUBLE PRECISION
The nonzero component of the rotated vector.

This version has a few statements commented out for thread safety
(machine parameters are computed on each entry). 10 feb 03, SJH.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine dlartgp (double precision F, double precision G, double precision CS, double precision SN, double precision R)

DLARTGP generates a plane rotation so that the diagonal is nonnegative.

Purpose:

DLARTGP generates a plane rotation so that

[CS SN] . [F] = [R] where CS**2 + SN**2 = 1.
[-SN CS] [G] [0]

This is a slower, more accurate version of the Level 1 BLAS routine DROTG,
with the following other differences:
F and G are unchanged on return.
If G=0, then CS=(+/-)1 and SN=0.
If F=0 and (G .ne. 0), then CS=0 and SN=(+/-)1.

The sign is chosen so that R >= 0.

Parameters

F

F is DOUBLE PRECISION
The first component of vector to be rotated.

G

G is DOUBLE PRECISION
The second component of vector to be rotated.

CS

CS is DOUBLE PRECISION
The cosine of the rotation.

SN

SN is DOUBLE PRECISION
The sine of the rotation.

R

R is DOUBLE PRECISION
The nonzero component of the rotated vector.

This version has a few statements commented out for thread safety
(machine parameters are computed on each entry). 10 feb 03, SJH.

Author

Univ. of Tennessee

Univ. of California Berkeley



Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine dlaruv (integer, dimension(4) ISEED, integer N, double precision, dimension(n) X)

DLARUV returns a vector of n random real numbers from a uniform distribution.

Purpose:

DLARUV returns a vector of n random real numbers from a uniform (0,1) distribution (n <= 128).

This is an auxiliary routine called by DLARNV and ZLARNV.

Parameters

ISEED

ISEED is INTEGER array, dimension (4)
On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and ISEED(4) must be odd.
On exit, the seed is updated.

N

N is INTEGER
The number of random numbers to be generated. N <= 128.

X

X is DOUBLE PRECISION array, dimension (N)
The generated random numbers.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Further Details:

This routine uses a multiplicative congruential method with modulus 2**48 and multiplier 33952834046453 (see G.S.Fishman, 'Multiplicative congruential random number generators with modulus 2**b: an exhaustive analysis for b = 32 and a partial analysis for b = 48', Math. Comp. 189, pp 331-344, 1990).

48-bit integers are stored in 4 integer array elements with 12 bits per element. Hence the routine is portable across machines with integers of 32 bits or more.

subroutine dlas2 (double precision F, double precision G, double precision H, double precision SSMIN, double precision SSMAX)

DLAS2 computes singular values of a 2-by-2 triangular matrix.

Purpose:

DLAS2 computes the singular values of the 2-by-2 matrix
[F G]



[0 H].
On return, SSMIN is the smaller singular value and SSMAX is the larger singular value.

Parameters

F

F is DOUBLE PRECISION
The (1,1) element of the 2-by-2 matrix.

G

G is DOUBLE PRECISION
The (1,2) element of the 2-by-2 matrix.

H

H is DOUBLE PRECISION
The (2,2) element of the 2-by-2 matrix.

SSMIN

SSMIN is DOUBLE PRECISION
The smaller singular value.

SSMAX

SSMAX is DOUBLE PRECISION
The larger singular value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

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Further Details:

Barring over/underflow, all output quantities are correct to within a few units in the last place (ulps), even in the absence of a guard digit in addition/subtraction.

In IEEE arithmetic, the code works correctly if one matrix element is infinite.

Overflow will not occur unless the largest singular value itself overflows, or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.)

Underflow is harmless if underflow is gradual. Otherwise, results may correspond to a matrix modified by perturbations of size near the underflow threshold.

subroutine dlascl (character TYPE, integer KL, integer KU, double precision CFROM, double precision CTO, integer M, integer N, double precision, dimension(lda, *) A, integer LDA, integer INFO)

DLASCL multiplies a general rectangular matrix by a real scalar defined as cto/cfrom.

Purpose:

DLASCL multiplies the M by N real matrix A by the real scalar CTO/CFROM. This is done without over/underflow as long as the final



result CTO*A(I,J)/CFROM does not over/underflow. TYPE specifies that A may be full, upper triangular, lower triangular, upper Hessenberg, or banded.

Parameters

TYPE

TYPE is CHARACTER*1
TYPE indices the storage type of the input matrix.
= 'G': A is a full matrix.
= 'L': A is a lower triangular matrix.
= 'U': A is an upper triangular matrix.
= 'H': A is an upper Hessenberg matrix.
= 'B': A is a symmetric band matrix with lower bandwidth KL and upper bandwidth KU and with the only the lower half stored.
= 'Q': A is a symmetric band matrix with lower bandwidth KL and upper bandwidth KU and with the only the upper half stored.
= 'Z': A is a band matrix with lower bandwidth KL and upper bandwidth KU. See DGBTRF for storage details.

KL

KL is INTEGER
The lower bandwidth of A. Referenced only if TYPE = 'B', 'Q' or 'Z'.

KU

KU is INTEGER
The upper bandwidth of A. Referenced only if TYPE = 'B', 'Q' or 'Z'.

CFROM

CFROM is DOUBLE PRECISION

CTO

CTO is DOUBLE PRECISION

The matrix A is multiplied by CTO/CFROM. A(I,J) is computed without over/underflow if the final result CTO*A(I,J)/CFROM can be represented without over/underflow. CFROM must be nonzero.

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

A

A is DOUBLE PRECISION array, dimension (LDA,N)
The matrix to be multiplied by CTO/CFROM. See TYPE for the storage type.

LDA

LDA is INTEGER
The leading dimension of the array A.
If TYPE = 'G', 'L', 'U', 'H', LDA >= max(1,M);
TYPE = 'B', LDA >= KL+1;



TYPE = 'Q', LDA >= KU+1;
TYPE = 'Z', LDA >= 2*KL+KU+1.

INFO

INFO is INTEGER
0 - successful exit
<0 - if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

subroutine dlasd0 (integer N, integer SQRE, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldvt, *) VT, integer LDVT, integer SMLSIZ, integer, dimension(*) IWORK, double precision, dimension(*) WORK, integer INFO)

DLASD0 computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

Using a divide and conquer approach, DLASD0 computes the singular value decomposition (SVD) of a real upper bidiagonal N-by-M matrix B with diagonal D and offdiagonal E, where M = N + SQRE. The algorithm computes orthogonal matrices U and VT such that B = U * S * VT. The singular values S are overwritten on D.

A related subroutine, DLASDA, computes only the singular values, and optionally, the singular vectors in compact form.

Parameters

N

N is INTEGER
On entry, the row dimension of the upper bidiagonal matrix. This is also the dimension of the main diagonal array D.

SQRE

SQRE is INTEGER
Specifies the column dimension of the bidiagonal matrix.
= 0: The bidiagonal matrix has column dimension M = N;
= 1: The bidiagonal matrix has column dimension M = N+1;

D

D is DOUBLE PRECISION array, dimension (N)
On entry D contains the main diagonal of the bidiagonal matrix.
On exit D, if INFO = 0, contains its singular values.

E

E is DOUBLE PRECISION array, dimension (M-1)
Contains the subdiagonal entries of the bidiagonal matrix.
On exit, E has been destroyed.

U

U is DOUBLE PRECISION array, dimension (LDU, N)
On exit, U contains the left singular vectors.



LDU

LDU is INTEGER
On entry, leading dimension of U.

VT

VT is DOUBLE PRECISION array, dimension (LDVT, M)
On exit, VT**T contains the right singular vectors.

LDVT

LDVT is INTEGER
On entry, leading dimension of VT.

SMLSIZ

SMLSIZ is INTEGER
On entry, maximum size of the subproblems at the
bottom of the computation tree.

IWORK

IWORK is INTEGER array, dimension (8*N)

WORK

WORK is DOUBLE PRECISION array, dimension (3*M**2+2*M)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd1 (integer NL, integer NR, integer SQRE, double precision, dimension(*) D, double precision ALPHA, double precision BETA, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldvt, *) VT, integer LDVT, integer, dimension(*) IDXQ, integer, dimension(*) IWORK, double precision, dimension(*) WORK, integer INFO)

DLASD1 computes the SVD of an upper bidiagonal matrix B of the specified size. Used by sbdsdc.

Purpose:

DLASD1 computes the SVD of an upper bidiagonal N-by-M matrix B,
where N = NL + NR + 1 and M = N + SQRE. DLASD1 is called from DLASD0.

A related subroutine DLASD7 handles the case in which the singular
values (and the singular vectors in factored form) are desired.

DLASD1 computes the SVD as follows:

```
          ( D1(in)    0    0       0 )
B = U(in) * (   Z1**T   a   Z2**T    b ) * VT(in)
          (   0       0   D2(in)   0 )
```



= U(out) * (D(out) 0) * VT(out)

where Z**T = (Z1**T a Z2**T b) = u**T VT**T, and u is a vector of dimension M with ALPHA and BETA in the NL+1 and NL+2 th entries and zeros elsewhere; and the entry b is empty if SQRE = 0.

The left singular vectors of the original matrix are stored in U, and the transpose of the right singular vectors are stored in VT, and the singular values are in D. The algorithm consists of three stages:

The first stage consists of deflating the size of the problem when there are multiple singular values or when there are zeros in the Z vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine DLASD2.

The second stage consists of calculating the updated singular values. This is done by finding the square roots of the roots of the secular equation via the routine DLASD4 (as called by DLASD3). This routine also calculates the singular vectors of the current problem.

The final stage consists of computing the updated singular vectors directly using the updated singular values. The singular vectors for the current problem are multiplied with the singular vectors from the overall problem.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has row dimension N = NL + NR + 1, and column dimension M = N + SQRE.

D

D is DOUBLE PRECISION array,
dimension (N = NL+NR+1).
On entry D(1:NL,1:NL) contains the singular values of the upper block; and D(NL+2:N) contains the singular values of the lower block. On exit D(1:N) contains the singular values of the modified matrix.

ALPHA

ALPHA is DOUBLE PRECISION
Contains the diagonal element associated with the added row.

BETA

BETA is DOUBLE PRECISION
Contains the off-diagonal element associated with the added row.



U

U is DOUBLE PRECISION array, dimension(LDU,N)
On entry U(1:NL, 1:NL) contains the left singular vectors of the upper block; U(NL+2:N, NL+2:N) contains the left singular vectors of the lower block. On exit U contains the left singular vectors of the bidiagonal matrix.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= max(1, N).

VT

VT is DOUBLE PRECISION array, dimension(LDVT,M)
where M = N + SQRE.
On entry VT(1:NL+1, 1:NL+1)**T contains the right singular vectors of the upper block; VT(NL+2:M, NL+2:M)**T contains the right singular vectors of the lower block. On exit VT**T contains the right singular vectors of the bidiagonal matrix.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= max(1, M).

IDXQ

IDXQ is INTEGER array, dimension(N)
This contains the permutation which will reintegrate the subproblem just solved back into sorted order, i.e.
D(IDXQ(I = 1, N)) will be in ascending order.

IWORK

IWORK is INTEGER array, dimension(4 * N)

WORK

WORK is DOUBLE PRECISION array, dimension(3*M**2 + 2*M)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd2 (integer NL, integer NR, integer SQRE, integer K, double precision, dimension(*) D, double precision, dimension(*) Z, double precision ALPHA, double precision BETA, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldvt, *) VT, integer LDVT, double precision, dimension(*) DSIGMA, double precision, dimension(ldu2, *) U2, integer LDU2, double precision, dimension(ldvt2, *) VT2, integer LDVT2, integer, dimension(*) IDXP, integer, dimension(*) IDX, integer, dimension(*) IDXC, integer, dimension(*) IDXQ, integer, dimension(*) COLTYP, integer INFO)



DLASD2 merges the two sets of singular values together into a single sorted set. Used by sbdsdc.

Purpose:

DLASD2 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more singular values are close together or if there is a tiny entry in the Z vector. For each such occurrence the order of the related secular equation problem is reduced by one.

DLASD2 is called from DLASD1.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has N = NL + NR + 1 rows and M = N + SQRE >= N columns.

K

K is INTEGER
Contains the dimension of the non-deflated matrix, This is the order of the related secular equation. 1 <= K <=N.

D

D is DOUBLE PRECISION array, dimension(N)
On entry D contains the singular values of the two submatrices to be combined. On exit D contains the trailing (N-K) updated singular values (those which were deflated) sorted into increasing order.

Z

Z is DOUBLE PRECISION array, dimension(N)
On exit Z contains the updating row vector in the secular equation.

ALPHA

ALPHA is DOUBLE PRECISION
Contains the diagonal element associated with the added row.

BETA

BETA is DOUBLE PRECISION
Contains the off-diagonal element associated with the added row.

U

U is DOUBLE PRECISION array, dimension(LDU,N)
On entry U contains the left singular vectors of two submatrices in the two square blocks with corners at (1,1), (NL, NL), and (NL+2, NL+2), (N,N).



On exit U contains the trailing (N-K) updated left singular
vectors (those which were deflated) in its last N-K columns.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= N.

VT

VT is DOUBLE PRECISION array, dimension(LDVT,M)
On entry VT**T contains the right singular vectors of two
submatrices in the two square blocks with corners at (1,1),
(NL+1, NL+1), and (NL+2, NL+2), (M,M).
On exit VT**T contains the trailing (N-K) updated right singular
vectors (those which were deflated) in its last N-K columns.
In case SQRE =1, the last row of VT spans the right null
space.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= M.

DSIGMA

DSIGMA is DOUBLE PRECISION array, dimension (N)
Contains a copy of the diagonal elements (K-1 singular values
and one zero) in the secular equation.

U2

U2 is DOUBLE PRECISION array, dimension(LDU2,N)
Contains a copy of the first K-1 left singular vectors which
will be used by DLASD3 in a matrix multiply (DGEMM) to solve
for the new left singular vectors. U2 is arranged into four
blocks. The first block contains a column with 1 at NL+1 and
zero everywhere else; the second block contains non-zero
entries only at and above NL; the third contains non-zero
entries only below NL+1; and the fourth is dense.

LDU2

LDU2 is INTEGER
The leading dimension of the array U2. LDU2 >= N.

VT2

VT2 is DOUBLE PRECISION array, dimension(LDVT2,N)
VT2**T contains a copy of the first K right singular vectors
which will be used by DLASD3 in a matrix multiply (DGEMM) to
solve for the new right singular vectors. VT2 is arranged into
three blocks. The first block contains a row that corresponds
to the special 0 diagonal element in SIGMA; the second block
contains non-zeros only at and before NL +1; the third block
contains non-zeros only at and after NL +2.

LDVT2

LDVT2 is INTEGER
The leading dimension of the array VT2. LDVT2 >= M.

IDXP

IDXP is INTEGER array, dimension(N)
This will contain the permutation used to place deflated
values of D at the end of the array. On output IDXP(2:K)
points to the nondeflated D-values and IDXP(K+1:N)
points to the deflated singular values.



IDX

IDX is INTEGER array, dimension(N)
This will contain the permutation used to sort the contents of D into ascending order.

IDXC

IDXC is INTEGER array, dimension(N)
This will contain the permutation used to arrange the columns of the deflated U matrix into three groups: the first group contains non-zero entries only at and above NL, the second contains non-zero entries only below NL+2, and the third is dense.

IDXQ

IDXQ is INTEGER array, dimension(N)
This contains the permutation which separately sorts the two sub-problems in D into ascending order. Note that entries in the first hlaf of this permutation must first be moved one position backward; and entries in the second half must first have NL+1 added to their values.

COLTYP

COLTYP is INTEGER array, dimension(N)
As workspace, this will contain a label which will indicate which of the following types a column in the U2 matrix or a row in the VT2 matrix is:
1 : non-zero in the upper half only
2 : non-zero in the lower half only
3 : dense
4 : deflated

On exit, it is an array of dimension 4, with COLTYP(I) being the dimension of the I-th type columns.

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd3 (integer NL, integer NR, integer SQRE, integer K, double precision, dimension(*) D, double precision, dimension(ldq, *) Q, integer LDQ, double precision, dimension(*) DSIGMA, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldu2, *) U2, integer LDU2, double precision, dimension(ldvt, *) VT, integer LDVT, double precision, dimension(ldvt2, *) VT2, integer LDVT2, integer, dimension(*) IDXC, integer, dimension(*) CTOT, double precision, dimension(*) Z, integer INFO)

DLASD3 finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by sbdsdc.

Purpose:



DLASD3 finds all the square roots of the roots of the secular equation, as defined by the values in D and Z. It makes the appropriate calls to DLASD4 and then updates the singular vectors by matrix multiplication.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray XMP, Cray YMP, Cray C 90, or Cray 2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

DLASD3 is called from DLASD1.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

K

K is INTEGER
The size of the secular equation, 1 =< K = < N.

D

D is DOUBLE PRECISION array, dimension(K)
On exit the square roots of the roots of the secular equation,
in ascending order.

Q

Q is DOUBLE PRECISION array, dimension (LDQ,K)

LDQ

LDQ is INTEGER
The leading dimension of the array Q. LDQ >= K.

DSIGMA

DSIGMA is DOUBLE PRECISION array, dimension(K)
The first K elements of this array contain the old roots
of the deflated updating problem. These are the poles
of the secular equation.

U

U is DOUBLE PRECISION array, dimension (LDU, N)
The last N - K columns of this matrix contain the deflated
left singular vectors.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= N.



U2

U2 is DOUBLE PRECISION array, dimension (LDU2, N)
The first K columns of this matrix contain the non-deflated
left singular vectors for the split problem.

LDU2

LDU2 is INTEGER
The leading dimension of the array U2. LDU2 >= N.

VT

VT is DOUBLE PRECISION array, dimension (LDVT, M)
The last M - K columns of VT**T contain the deflated
right singular vectors.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= N.

VT2

VT2 is DOUBLE PRECISION array, dimension (LDVT2, N)
The first K columns of VT2**T contain the non-deflated
right singular vectors for the split problem.

LDVT2

LDVT2 is INTEGER
The leading dimension of the array VT2. LDVT2 >= N.

IDXC

IDXC is INTEGER array, dimension (N)
The permutation used to arrange the columns of U (and rows of
VT) into three groups: the first group contains non-zero
entries only at and above (or before) NL +1; the second
contains non-zero entries only at and below (or after) NL+2;
and the third is dense. The first column of U and the row of
VT are treated separately, however.

The rows of the singular vectors found by DLASD4
must be likewise permuted before the matrix multiplies can
take place.

CTOT

CTOT is INTEGER array, dimension (4)
A count of the total number of the various types of columns
in U (or rows in VT), as described in IDXC. The fourth column
type is any column which has been deflated.

Z

Z is DOUBLE PRECISION array, dimension (K)
The first K elements of this array contain the components
of the deflation-adjusted updating row vector.

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley



Univ. of Colorado Denver

NAG Ltd.

Date
June 2017

Contributors:
Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd4 (integer N, integer I, double precision, dimension(*) D, double precision, dimension(*) Z, double precision, dimension(*) DELTA, double precision RHO, double precision SIGMA, double precision, dimension(*) WORK, integer INFO)
DLASD4 computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by dbdsdc.

Purpose:

This subroutine computes the square root of the I-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix whose entries are given as the squares of the corresponding entries in the array d, and that

0 <= D(i) < D(j) for i < j

and that RHO > 0. This is arranged by the calling routine, and is no loss in generality. The rank-one modified system is thus

diag(D) * diag(D) + RHO * Z * Z_transpose.

where we assume the Euclidean norm of Z is 1.

The method consists of approximating the rational functions in the secular equation by simpler interpolating rational functions.

Parameters

N

N is INTEGER
The length of all arrays.

I

I is INTEGER
The index of the eigenvalue to be computed. 1 <= I <= N.

D

D is DOUBLE PRECISION array, dimension (N)
The original eigenvalues. It is assumed that they are in order, 0 <= D(I) < D(J) for I < J.

Z

Z is DOUBLE PRECISION array, dimension (N)
The components of the updating vector.

DELTA

DELTA is DOUBLE PRECISION array, dimension (N)
If N .ne. 1, DELTA contains (D(j) - sigma_I) in its j-th component. If N = 1, then DELTA(1) = 1. The vector DELTA contains the information necessary to construct the (singular) eigenvectors.

RHO

RHO is DOUBLE PRECISION



The scalar in the symmetric updating formula.

SIGMA

SIGMA is DOUBLE PRECISION
The computed sigma_I, the I-th updated eigenvalue.

WORK

WORK is DOUBLE PRECISION array, dimension (N)
If N .ne. 1, WORK contains (D(j) + sigma_I) in its j-th component. If N = 1, then WORK(1) = 1.

INFO

INFO is INTEGER
= 0: successful exit
> 0: if INFO = 1, the updating process failed.

Internal Parameters:

Logical variable ORGATI (origin-at-i?) is used for distinguishing whether D(i) or D(i+1) is treated as the origin.

ORGATI = .true. origin at i
ORGATI = .false. origin at i+1

Logical variable SWTCH3 (switch-for-3-poles?) is for noting if we are working with THREE poles!

MAXIT is the maximum number of iterations allowed for each eigenvalue.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd5 (integer I, double precision, dimension(2) D, double precision, dimension(2) Z, double precision, dimension(2) DELTA, double precision RHO, double precision DSIGMA, double precision, dimension(2) WORK)

DLASD5 computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix. Used by sbdsdc.

Purpose:

This subroutine computes the square root of the I-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix

diag(D) * diag(D) + RHO * Z * transpose(Z) .

The diagonal entries in the array D are assumed to satisfy

0 <= D(i) < D(j) for i < j .

We also assume RHO > 0 and that the Euclidean norm of the vector



Z is one.

Parameters

I

I is INTEGER
The index of the eigenvalue to be computed. I = 1 or I = 2.

D

D is DOUBLE PRECISION array, dimension (2)
The original eigenvalues. We assume 0 <= D(1) < D(2).

Z

Z is DOUBLE PRECISION array, dimension (2)
The components of the updating vector.

DELTA

DELTA is DOUBLE PRECISION array, dimension (2)
Contains (D(j) - sigma_I) in its j-th component.
The vector DELTA contains the information necessary
to construct the eigenvectors.

RHO

RHO is DOUBLE PRECISION
The scalar in the symmetric updating formula.

DSIGMA

DSIGMA is DOUBLE PRECISION
The computed sigma_I, the I-th updated eigenvalue.

WORK

WORK is DOUBLE PRECISION array, dimension (2)
WORK contains (D(j) + sigma_I) in its j-th component.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd6 (integer ICOMPQ, integer NL, integer NR, integer SQRE, double precision, dimension(*) D, double precision, dimension(*) VF, double precision, dimension(*) VL, double precision ALPHA, double precision BETA, integer, dimension(*) IDXQ, integer, dimension(*) PERM, integer GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, double precision, dimension(ldgnum, *) GIVNUM, integer LDGNUM, double precision, dimension(ldgnum, *) POLES, double precision, dimension(*) DIFL, double precision, dimension(*) DIFR, double precision, dimension(*) Z, integer K, double precision C, double precision S, double precision, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

DLASD6 computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by sbdsdc.

Purpose:

DLASD6 computes the SVD of an updated upper bidiagonal matrix B
obtained by merging two smaller ones by appending a row. This



routine is used only for the problem which requires all singular values and optionally singular vector matrices in factored form. B is an N-by-M matrix with N = NL + NR + 1 and M = N + SQRE. A related subroutine, DLASD1, handles the case in which all singular values and singular vectors of the bidiagonal matrix are desired.

DLASD6 computes the SVD as follows:

```
            ( D1(in)    0    0       0 )
 B = U(in) * (   Z1**T   a   Z2**T    b ) * VT(in)
            (   0       0   D2(in)   0 )

   = U(out) * ( D(out) 0) * VT(out)
```

where Z**T = (Z1**T a Z2**T b) = u**T VT**T, and u is a vector of dimension M with ALPHA and BETA in the NL+1 and NL+2 th entries and zeros elsewhere; and the entry b is empty if SQRE = 0.

The singular values of B can be computed using D1, D2, the first components of all the right singular vectors of the lower block, and the last components of all the right singular vectors of the upper block. These components are stored and updated in VF and VL, respectively, in DLASD6. Hence U and VT are not explicitly referenced.

The singular values are stored in D. The algorithm consists of two stages:

The first stage consists of deflating the size of the problem when there are multiple singular values or if there is a zero in the Z vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine DLASD7.

The second stage consists of calculating the updated singular values. This is done by finding the roots of the secular equation via the routine DLASD4 (as called by DLASD8). This routine also updates VF and VL and computes the distances between the updated singular values and the old singular values.

DLASD6 is called from DLASDA.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in factored form:
= 0: Compute singular values only.
= 1: Compute singular vectors in factored form as well.

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE



SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has row dimension N = NL + NR + 1,
and column dimension M = N + SQRE.

D

D is DOUBLE PRECISION array, dimension (NL+NR+1).
On entry D(1:NL,1:NL) contains the singular values of the
upper block, and D(NL+2:N) contains the singular values
of the lower block. On exit D(1:N) contains the singular
values of the modified matrix.

VF

VF is DOUBLE PRECISION array, dimension (M)
On entry, VF(1:NL+1) contains the first components of all
right singular vectors of the upper block; and VF(NL+2:M)
contains the first components of all right singular vectors
of the lower block. On exit, VF contains the first components
of all right singular vectors of the bidiagonal matrix.

VL

VL is DOUBLE PRECISION array, dimension (M)
On entry, VL(1:NL+1) contains the last components of all
right singular vectors of the upper block; and VL(NL+2:M)
contains the last components of all right singular vectors of
the lower block. On exit, VL contains the last components of
all right singular vectors of the bidiagonal matrix.

ALPHA

ALPHA is DOUBLE PRECISION
Contains the diagonal element associated with the added row.

BETA

BETA is DOUBLE PRECISION
Contains the off-diagonal element associated with the added
row.

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which will reintegrate the
subproblem just solved back into sorted order, i.e.
D(IDXQ(I = 1, N)) will be in ascending order.

PERM

PERM is INTEGER array, dimension (N)
The permutations (from deflation and sorting) to be applied
to each block. Not referenced if ICOMPQ = 0.

GIVPTR

GIVPTR is INTEGER
The number of Givens rotations which took place in this
subproblem. Not referenced if ICOMPQ = 0.

GIVCOL

GIVCOL is INTEGER array, dimension (LDGCOL, 2)
Each pair of numbers indicates a pair of columns to take place
in a Givens rotation. Not referenced if ICOMPQ = 0.

LDGCOL



LDGCOL is INTEGER
leading dimension of GIVCOL, must be at least N.

GIVNUM

GIVNUM is DOUBLE PRECISION array, dimension (LDGNUM, 2)
Each number indicates the C or S value to be used in the
corresponding Givens rotation. Not referenced if ICOMPQ = 0.

LDGNUM

LDGNUM is INTEGER
The leading dimension of GIVNUM and POLES, must be at least N.

POLES

POLES is DOUBLE PRECISION array, dimension (LDGNUM, 2)
On exit, POLES(1,*) is an array containing the new singular
values obtained from solving the secular equation, and
POLES(2,*) is an array containing the poles in the secular
equation. Not referenced if ICOMPQ = 0.

DIFL

DIFL is DOUBLE PRECISION array, dimension (N)
On exit, DIFL(I) is the distance between I-th updated
(undeflated) singular value and the I-th (undeflated) old
singular value.

DIFR

DIFR is DOUBLE PRECISION array,
dimension (LDDIFR, 2) if ICOMPQ = 1 and
dimension (K) if ICOMPQ = 0.
On exit, DIFR(I,1) = D(I) - DSIGMA(I+1), DIFR(K,1) is not
defined and will not be referenced.

If ICOMPQ = 1, DIFR(1:K,2) is an array containing the
normalizing factors for the right singular vector matrix.

See DLASD8 for details on DIFL and DIFR.

Z

Z is DOUBLE PRECISION array, dimension (M)
The first elements of this array contain the components
of the deflation-adjusted updating row vector.

K

K is INTEGER
Contains the dimension of the non-deflated matrix,
This is the order of the related secular equation. 1 <= K <=N.

C

C is DOUBLE PRECISION
C contains garbage if SQRE =0 and the C-value of a Givens
rotation related to the right null space if SQRE = 1.

S

S is DOUBLE PRECISION
S contains garbage if SQRE =0 and the S-value of a Givens
rotation related to the right null space if SQRE = 1.

WORK

WORK is DOUBLE PRECISION array, dimension (4 * M)

IWORK



IWORK is INTEGER array, dimension (3 * N)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd7 (integer ICOMPQ, integer NL, integer NR, integer SQRE, integer K, double precision, dimension(*) D, double precision, dimension(*) Z, double precision, dimension(*) ZW, double precision, dimension(*) VF, double precision, dimension(*) VFW, double precision, dimension(*) VL, double precision, dimension(*) VLW, double precision ALPHA, double precision BETA, double precision, dimension(*) DSIGMA, integer, dimension(*) IDX, integer, dimension(*) IDXP, integer, dimension(*) IDXQ, integer, dimension(*) PERM, integer GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, double precision, dimension(ldgnum, *) GIVNUM, integer LDGNUM, double precision C, double precision S, integer INFO)

DLASD7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by sbdsdc.

Purpose:

DLASD7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more singular values are close together or if there is a tiny entry in the Z vector. For each such occurrence the order of the related secular equation problem is reduced by one.

DLASD7 is called from DLASD6.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in compact form, as follows:
= 0: Compute singular values only.
= 1: Compute singular vectors of upper bidiagonal matrix in compact form.

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE



SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has
N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

K

K is INTEGER
Contains the dimension of the non-deflated matrix, this is the order of the related secular equation. 1 <= K <=N.

D

D is DOUBLE PRECISION array, dimension (N)
On entry D contains the singular values of the two submatrices to be combined. On exit D contains the trailing (N-K) updated singular values (those which were deflated) sorted into increasing order.

Z

Z is DOUBLE PRECISION array, dimension (M)
On exit Z contains the updating row vector in the secular equation.

ZW

ZW is DOUBLE PRECISION array, dimension (M)
Workspace for Z.

VF

VF is DOUBLE PRECISION array, dimension (M)
On entry, VF(1:NL+1) contains the first components of all right singular vectors of the upper block; and VF(NL+2:M) contains the first components of all right singular vectors of the lower block. On exit, VF contains the first components of all right singular vectors of the bidiagonal matrix.

VFW

VFW is DOUBLE PRECISION array, dimension (M)
Workspace for VF.

VL

VL is DOUBLE PRECISION array, dimension (M)
On entry, VL(1:NL+1) contains the last components of all right singular vectors of the upper block; and VL(NL+2:M) contains the last components of all right singular vectors of the lower block. On exit, VL contains the last components of all right singular vectors of the bidiagonal matrix.

VLW

VLW is DOUBLE PRECISION array, dimension (M)
Workspace for VL.

ALPHA

ALPHA is DOUBLE PRECISION
Contains the diagonal element associated with the added row.

BETA

BETA is DOUBLE PRECISION
Contains the off-diagonal element associated with the added row.



DSIGMA

DSIGMA is DOUBLE PRECISION array, dimension (N)
Contains a copy of the diagonal elements (K-1 singular values and one zero) in the secular equation.

IDX

IDX is INTEGER array, dimension (N)
This will contain the permutation used to sort the contents of D into ascending order.

IDXP

IDXP is INTEGER array, dimension (N)
This will contain the permutation used to place deflated values of D at the end of the array. On output IDXP(2:K) points to the nondeflated D-values and IDXP(K+1:N) points to the deflated singular values.

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which separately sorts the two sub-problems in D into ascending order. Note that entries in the first half of this permutation must first be moved one position backward; and entries in the second half must first have NL+1 added to their values.

PERM

PERM is INTEGER array, dimension (N)
The permutations (from deflation and sorting) to be applied to each singular block. Not referenced if ICOMPQ = 0.

GIVPTR

GIVPTR is INTEGER
The number of Givens rotations which took place in this subproblem. Not referenced if ICOMPQ = 0.

GIVCOL

GIVCOL is INTEGER array, dimension (LDGCOL, 2)
Each pair of numbers indicates a pair of columns to take place in a Givens rotation. Not referenced if ICOMPQ = 0.

LDGCOL

LDGCOL is INTEGER
The leading dimension of GIVCOL, must be at least N.

GIVNUM

GIVNUM is DOUBLE PRECISION array, dimension (LDGNUM, 2)
Each number indicates the C or S value to be used in the corresponding Givens rotation. Not referenced if ICOMPQ = 0.

LDGNUM

LDGNUM is INTEGER
The leading dimension of GIVNUM, must be at least N.

C

C is DOUBLE PRECISION
C contains garbage if SQRE =0 and the C-value of a Givens rotation related to the right null space if SQRE = 1.

S

S is DOUBLE PRECISION



S contains garbage if SQRE =0 and the S-value of a Givens
rotation related to the right null space if SQRE = 1.

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasd8 (integer ICOMPQ, integer K, double precision, dimension(*) D, double precision, dimension(*) Z, double precision, dimension(*) VF, double precision, dimension(*) VL, double precision, dimension(*) DIFL, double precision, dimension(lddifr, *) DIFR, integer LDDIFR, double precision, dimension(*) DSIGMA, double precision, dimension(*) WORK, integer INFO)

DLASD8 finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by sbdsdc.

Purpose:

DLASD8 finds the square roots of the roots of the secular equation,
as defined by the values in DSIGMA and Z. It makes the appropriate
calls to DLASD4, and stores, for each element in D, the distance
to its two nearest poles (elements in DSIGMA). It also updates
the arrays VF and VL, the first and last components of all the
right singular vectors of the original bidiagonal matrix.

DLASD8 is called from DLASD6.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in
factored form in the calling routine:
= 0: Compute singular values only.
= 1: Compute singular vectors in factored form as well.

K

K is INTEGER
The number of terms in the rational function to be solved
by DLASD4. K >= 1.

D

D is DOUBLE PRECISION array, dimension (K)
On output, D contains the updated singular values.

Z

Z is DOUBLE PRECISION array, dimension (K)
On entry, the first K elements of this array contain the
components of the deflation-adjusted updating row vector.
On exit, Z is updated.



VF

VF is DOUBLE PRECISION array, dimension (K)
On entry, VF contains information passed through DBEDE8.
On exit, VF contains the first K components of the first components of all right singular vectors of the bidiagonal matrix.

VL

VL is DOUBLE PRECISION array, dimension (K)
On entry, VL contains information passed through DBEDE8.
On exit, VL contains the first K components of the last components of all right singular vectors of the bidiagonal matrix.

DIFL

DIFL is DOUBLE PRECISION array, dimension (K)
On exit, DIFL(I) = D(I) - DSIGMA(I).

DIFR

DIFR is DOUBLE PRECISION array,
dimension (LDDIFR, 2) if ICOMPQ = 1 and
dimension (K) if ICOMPQ = 0.
On exit, DIFR(I,1) = D(I) - DSIGMA(I+1), DIFR(K,1) is not defined and will not be referenced.

If ICOMPQ = 1, DIFR(1:K,2) is an array containing the normalizing factors for the right singular vector matrix.

LDDIFR

LDDIFR is INTEGER
The leading dimension of DIFR, must be at least K.

DSIGMA

DSIGMA is DOUBLE PRECISION array, dimension (K)
On entry, the first K elements of this array contain the old roots of the deflated updating problem. These are the poles of the secular equation.
On exit, the elements of DSIGMA may be very slightly altered in value.

WORK

WORK is DOUBLE PRECISION array, dimension (3*K)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA



subroutine dlasda (integer ICOMPQ, integer SMLSIZ, integer N, integer SQRE, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldu, *) VT, integer, dimension(*) K, double precision, dimension(ldu, *) DIFL, double precision, dimension(ldu, *) DIFR, double precision, dimension(ldu, *) Z, double precision, dimension(ldu, *) POLES, integer, dimension(*) GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, integer, dimension(ldgcol, *) PERM, double precision, dimension(ldu, *) GIVNUM, double precision, dimension(*) C, double precision, dimension(*) S, double precision, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

DLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

Using a divide and conquer approach, DLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal N-by-M matrix B with diagonal D and offdiagonal E, where M = N + SQRE. The algorithm computes the singular values in the SVD B = U * S * VT. The orthogonal matrices U and VT are optionally computed in compact form.

A related subroutine, DLASD0, computes the singular values and the singular vectors in explicit form.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in compact form, as follows
= 0: Compute singular values only.
= 1: Compute singular vectors of upper bidiagonal matrix in compact form.

SMLSIZ

SMLSIZ is INTEGER
The maximum size of the subproblems at the bottom of the computation tree.

N

N is INTEGER
The row dimension of the upper bidiagonal matrix. This is also the dimension of the main diagonal array D.

SQRE

SQRE is INTEGER
Specifies the column dimension of the bidiagonal matrix.
= 0: The bidiagonal matrix has column dimension M = N;
= 1: The bidiagonal matrix has column dimension M = N + 1.

D

D is DOUBLE PRECISION array, dimension (N)
On entry D contains the main diagonal of the bidiagonal matrix. On exit D, if INFO = 0, contains its singular values.

E

E is DOUBLE PRECISION array, dimension (M-1)
Contains the subdiagonal entries of the bidiagonal matrix. On exit, E has been destroyed.

U



U is DOUBLE PRECISION array,
dimension (LDU, SMLSIZ) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, U contains the left
singular vector matrices of all subproblems at the bottom
level.

LDU

LDU is INTEGER, LDU = > N.
The leading dimension of arrays U, VT, DIFL, DIFR, POLES,
GIVNUM, and Z.

VT

VT is DOUBLE PRECISION array,
dimension (LDU, SMLSIZ+1) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, VT**T contains the right
singular vector matrices of all subproblems at the bottom
level.

K

K is INTEGER array,
dimension (N) if ICOMPQ = 1 and dimension 1 if ICOMPQ = 0.
If ICOMPQ = 1, on exit, K(I) is the dimension of the I-th
secular equation on the computation tree.

DIFL

DIFL is DOUBLE PRECISION array, dimension (LDU, NLVL),
where NLVL = floor(log_2 (N/SMLSIZ))).

DIFR

DIFR is DOUBLE PRECISION array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1 and
dimension (N) if ICOMPQ = 0.
If ICOMPQ = 1, on exit, DIFL(1:N, I) and DIFR(1:N, 2 * I - 1)
record distances between singular values on the I-th
level and singular values on the (I -1)-th level, and
DIFR(1:N, 2 * I) contains the normalizing factors for
the right singular vector matrix. See DLASD8 for details.

Z

Z is DOUBLE PRECISION array,
dimension (LDU, NLVL) if ICOMPQ = 1 and
dimension (N) if ICOMPQ = 0.
The first K elements of Z(1, I) contain the components of
the deflation-adjusted updating row vector for subproblems
on the I-th level.

POLES

POLES is DOUBLE PRECISION array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, POLES(1, 2*I - 1) and
POLES(1, 2*I) contain the new and old singular values
involved in the secular equations on the I-th level.

GIVPTR

GIVPTR is INTEGER array,
dimension (N) if ICOMPQ = 1, and not referenced if
ICOMPQ = 0. If ICOMPQ = 1, on exit, GIVPTR(I) records
the number of Givens rotations performed on the I-th
problem on the computation tree.

GIVCOL



GIVCOL is INTEGER array,
dimension (LDGCOL, 2 * NLVL) if ICOMPQ = 1, and not
referenced if ICOMPQ = 0. If ICOMPQ = 1, on exit, for each I,
GIVCOL(1, 2 *I - 1) and GIVCOL(1, 2 *I) record the locations
of Givens rotations performed on the I-th level on the
computation tree.

LDGCOL

LDGCOL is INTEGER, LDGCOL = > N.
The leading dimension of arrays GIVCOL and PERM.

PERM

PERM is INTEGER array,
dimension (LDGCOL, NLVL) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, PERM(1, I) records
permutations done on the I-th level of the computation tree.

GIVNUM

GIVNUM is DOUBLE PRECISION array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1, and not
referenced if ICOMPQ = 0. If ICOMPQ = 1, on exit, for each I,
GIVNUM(1, 2 *I - 1) and GIVNUM(1, 2 *I) record the C- and S-
values of Givens rotations performed on the I-th level on
the computation tree.

C

C is DOUBLE PRECISION array,
dimension (N) if ICOMPQ = 1, and dimension 1 if ICOMPQ = 0.
If ICOMPQ = 1 and the I-th subproblem is not square, on exit,
C(I) contains the C-value of a Givens rotation related to
the right null space of the I-th subproblem.

S

S is DOUBLE PRECISION array, dimension (N) if
ICOMPQ = 1, and dimension 1 if ICOMPQ = 0. If ICOMPQ = 1
and the I-th subproblem is not square, on exit, S(I)
contains the S-value of a Givens rotation related to
the right null space of the I-th subproblem.

WORK

WORK is DOUBLE PRECISION array, dimension
(6 * N + (SMLSIZ + 1)*(SMLSIZ + 1)).

IWORK

IWORK is INTEGER array, dimension (7*N)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017



Contributors:
Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasdq (character UPLO, integer SQRE, integer N, integer NCVT, integer NRU, integer NCC, double precision, dimension(*) D, double precision, dimension(*) E, double precision, dimension(ldvt, *) VT, integer LDVT, double precision, dimension(ldu, *) U, integer LDU, double precision, dimension(ldc, *) C, integer LDC, double precision, dimension(*) WORK, integer INFO)

DLASDQ computes the SVD of a real bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

DLASDQ computes the singular value decomposition (SVD) of a real (upper or lower) bidiagonal matrix with diagonal D and offdiagonal E, accumulating the transformations if desired. Letting B denote the input bidiagonal matrix, the algorithm computes orthogonal matrices Q and P such that B = Q * S * P**T (P**T denotes the transpose of P). The singular values S are overwritten on D.

The input matrix U is changed to U * Q if desired.
The input matrix VT is changed to P**T * VT if desired.
The input matrix C is changed to Q**T * C if desired.

See "Computing Small Singular Values of Bidiagonal Matrices With Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan, LAPACK Working Note #3, for a detailed description of the algorithm.

Parameters

UPLO

UPLO is CHARACTER*1
On entry, UPLO specifies whether the input bidiagonal matrix is upper or lower bidiagonal, and whether it is square are not.
UPLO = 'U' or 'u' B is upper bidiagonal.
UPLO = 'L' or 'l' B is lower bidiagonal.

SQRE

SQRE is INTEGER
= 0: then the input matrix is N-by-N.
= 1: then the input matrix is N-by-(N+1) if UPLU = 'U' and (N+1)-by-N if UPLU = 'L'.

The bidiagonal matrix has
N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

N

N is INTEGER
On entry, N specifies the number of rows and columns in the matrix. N must be at least 0.

NCVT

NCVT is INTEGER
On entry, NCVT specifies the number of columns of the matrix VT. NCVT must be at least 0.

NRU

NRU is INTEGER
On entry, NRU specifies the number of rows of



the matrix U. NRU must be at least 0.

NCC

NCC is INTEGER
On entry, NCC specifies the number of columns of
the matrix C. NCC must be at least 0.

D

D is DOUBLE PRECISION array, dimension (N)
On entry, D contains the diagonal entries of the
bidiagonal matrix whose SVD is desired. On normal exit,
D contains the singular values in ascending order.

E

E is DOUBLE PRECISION array.
dimension is (N-1) if SQRE = 0 and N if SQRE = 1.
On entry, the entries of E contain the offdiagonal entries
of the bidiagonal matrix whose SVD is desired. On normal
exit, E will contain 0. If the algorithm does not converge,
D and E will contain the diagonal and superdiagonal entries
of a bidiagonal matrix orthogonally equivalent to the one
given as input.

VT

VT is DOUBLE PRECISION array, dimension (LDVT, NCVT)
On entry, contains a matrix which on exit has been
premultiplied by P**T, dimension N-by-NCVT if SQRE = 0
and (N+1)-by-NCVT if SQRE = 1 (not referenced if NCVT=0).

LDVT

LDVT is INTEGER
On entry, LDVT specifies the leading dimension of VT as
declared in the calling (sub) program. LDVT must be at
least 1. If NCVT is nonzero LDVT must also be at least N.

U

U is DOUBLE PRECISION array, dimension (LDU, N)
On entry, contains a matrix which on exit has been
postmultiplied by Q, dimension NRU-by-N if SQRE = 0
and NRU-by-(N+1) if SQRE = 1 (not referenced if NRU=0).

LDU

LDU is INTEGER
On entry, LDU specifies the leading dimension of U as
declared in the calling (sub) program. LDU must be at
least max(1, NRU) .

C

C is DOUBLE PRECISION array, dimension (LDC, NCC)
On entry, contains an N-by-NCC matrix which on exit
has been premultiplied by Q**T dimension N-by-NCC if SQRE = 0
and (N+1)-by-NCC if SQRE = 1 (not referenced if NCC=0).

LDC

LDC is INTEGER
On entry, LDC specifies the leading dimension of C as
declared in the calling (sub) program. LDC must be at
least 1. If NCC is nonzero, LDC must also be at least N.

WORK

WORK is DOUBLE PRECISION array, dimension (4*N)



Workspace. Only referenced if one of NCVT, NRU, or NCC is nonzero, and if N is at least 2.

INFO

INFO is INTEGER
On exit, a value of 0 indicates a successful exit.
If INFO < 0, argument number -INFO is illegal.
If INFO > 0, the algorithm did not converge, and INFO specifies how many superdiagonals did not converge.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlasdt (integer N, integer LVL, integer ND, integer, dimension(*) INODE, integer, dimension(*) NDIML, integer, dimension(*) NDIMR, integer MSUB)

DLASDT creates a tree of subproblems for bidiagonal divide and conquer. Used by sbdsdc.

Purpose:

DLASDT creates a tree of subproblems for bidiagonal divide and conquer.

Parameters

N

N is INTEGER
On entry, the number of diagonal elements of the bidiagonal matrix.

LVL

LVL is INTEGER
On exit, the number of levels on the computation tree.

ND

ND is INTEGER
On exit, the number of nodes on the tree.

INODE

INODE is INTEGER array, dimension (N)
On exit, centers of subproblems.

NDIML

NDIML is INTEGER array, dimension (N)
On exit, row dimensions of left children.

NDIMR

NDIMR is INTEGER array, dimension (N)
On exit, row dimensions of right children.

MSUB

MSUB is INTEGER
On entry, the maximum row dimension each subproblem at the bottom of the tree can be of.



Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine dlaset (character UPLO, integer M, integer N, double precision ALPHA, double precision BETA, double precision, dimension(lda, *) A, integer LDA)

DLASET initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

Purpose:

DLASET initializes an m-by-n matrix A to BETA on the diagonal and ALPHA on the offdiagonals.

Parameters

UPLO

UPLO is CHARACTER*1
Specifies the part of the matrix A to be set.
= 'U': Upper triangular part is set; the strictly lower triangular part of A is not changed.
= 'L': Lower triangular part is set; the strictly upper triangular part of A is not changed.
Otherwise: All of the matrix A is set.

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

ALPHA

ALPHA is DOUBLE PRECISION
The constant to which the offdiagonal elements are to be set.

BETA

BETA is DOUBLE PRECISION
The constant to which the diagonal elements are to be set.

A

A is DOUBLE PRECISION array, dimension (LDA,N)
On exit, the leading m-by-n submatrix of A is set as follows:

if UPLO = 'U', A(i,j) = ALPHA, 1<=i<=j-1, 1<=j<=n,
if UPLO = 'L', A(i,j) = ALPHA, j+1<=i<=m, 1<=j<=n,
otherwise, A(i,j) = ALPHA, 1<=i<=m, 1<=j<=n, i.ne.j,

and, for all UPLO, A(i,i) = BETA, 1<=i<=min(m,n).

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).



Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine dlasr (character SIDE, character PIVOT, character DIRECT, integer M, integer N, double precision, dimension(*) C, double precision, dimension(*) S, double precision, dimension(lda, *) A, integer LDA)

DLASR applies a sequence of plane rotations to a general rectangular matrix.

Purpose:

DLASR applies a sequence of plane rotations to a real matrix A,
from either the left or the right.

When SIDE = 'L', the transformation takes the form

A := P*A

and when SIDE = 'R', the transformation takes the form

A := A*P**T

where P is an orthogonal matrix consisting of a sequence of z plane
rotations, with z = M when SIDE = 'L' and z = N when SIDE = 'R',
and P**T is the transpose of P.

When DIRECT = 'F' (Forward sequence), then

P = P(z-1) * ... * P(2) * P(1)

and when DIRECT = 'B' (Backward sequence), then

P = P(1) * P(2) * ... * P(z-1)

where P(k) is a plane rotation matrix defined by the 2-by-2 rotation

R(k) = (c(k) s(k))
= (-s(k) c(k)).

When PIVOT = 'V' (Variable pivot), the rotation is performed
for the plane (k,k+1), i.e., P(k) has the form

```
P(k) = (  1                                  )
       (      ...                            )
       (             1                       )
       (                c(k)  s(k)           )
       (               -s(k)  c(k)           )
       (                             1       )
       (                                ...  )
       (                                    1 )
```

where R(k) appears as a rank-2 modification to the identity matrix in
rows and columns k and k+1.



When PIVOT = 'T' (Top pivot), the rotation is performed for the plane (1,k+1), so P(k) has the form

```
P(k) = (  c(k)                 s(k)                )
       (        1                                  )
       (             ...                           )
       (                    1                      )
       ( -s(k)                c(k)                 )
       (                           1               )
       (                                ...        )
       (                                      1 )
```

where R(k) appears in rows and columns 1 and k+1.

Similarly, when PIVOT = 'B' (Bottom pivot), the rotation is performed for the plane (k,z), giving P(k) the form

```
P(k) = ( 1                                         )
       (     ...                                   )
       (           1                               )
       (              c(k)                 s(k) )
       (                    1                     )
       (                         ...              )
       (                              1           )
       (              -s(k)                c(k) )
```

where R(k) appears in rows and columns k and z. The rotations are performed without ever forming P(k) explicitly.

Parameters

SIDE

SIDE is CHARACTER*1
Specifies whether the plane rotation matrix P is applied to A on the left or the right.
= 'L': Left, compute A := P*A
= 'R': Right, compute A:= A*P**T

PIVOT

PIVOT is CHARACTER*1
Specifies the plane for which P(k) is a plane rotation matrix.
= 'V': Variable pivot, the plane (k,k+1)
= 'T': Top pivot, the plane (1,k+1)
= 'B': Bottom pivot, the plane (k,z)

DIRECT

DIRECT is CHARACTER*1
Specifies whether P is a forward or backward sequence of plane rotations.
= 'F': Forward, P = P(z-1)*...*P(2)*P(1)
= 'B': Backward, P = P(1)*P(2)*...*P(z-1)

M

M is INTEGER
The number of rows of the matrix A. If m <= 1, an immediate return is effected.

N

N is INTEGER
The number of columns of the matrix A. If n <= 1, an



immediate return is effected.

C

C is DOUBLE PRECISION array, dimension
(M-1) if SIDE = 'L'
(N-1) if SIDE = 'R'
The cosines c(k) of the plane rotations.

S

S is DOUBLE PRECISION array, dimension
(M-1) if SIDE = 'L'
(N-1) if SIDE = 'R'
The sines s(k) of the plane rotations. The 2-by-2 plane rotation part of the matrix P(k), R(k), has the form
R(k) = (c(k) s(k))
(-s(k) c(k)).

A

A is DOUBLE PRECISION array, dimension (LDA,N)
The M-by-N matrix A. On exit, A is overwritten by P*A if SIDE = 'L' or by A*P**T if SIDE = 'R'.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine dlassq (integer N, double precision, dimension(*) X, integer INCX, double precision SCALE, double precision SUMSQ)

DLASSQ updates a sum of squares represented in scaled form.

Purpose:

DLASSQ returns the values scl and smsq such that

(scl**2)*smsq = x(1)**2 +...+ x(n)**2 + (scale**2)*sumsq,

where x(i) = X(1 + (i - 1)*INCX). The value of sumsq is assumed to be non-negative and scl returns the value

scl = max(scale, abs(x(i))).

scale and sumsq must be supplied in SCALE and SUMSQ and scl and smsq are overwritten on SCALE and SUMSQ respectively.

The routine makes only one pass through the vector x.

Parameters

N

N is INTEGER
The number of elements to be used from the vector X.

X



X is DOUBLE PRECISION array, dimension (1+(N-1)*INCX)
The vector for which a scaled sum of squares is computed.
x(i) = X(1 + (i - 1)*INCX), 1 <= i <= n.

INCX

INCX is INTEGER
The increment between successive values of the vector X.
INCX > 0.

SCALE

SCALE is DOUBLE PRECISION
On entry, the value scale in the equation above.
On exit, SCALE is overwritten with scl , the scaling factor
for the sum of squares.

SUMSQ

SUMSQ is DOUBLE PRECISION
On entry, the value sumsq in the equation above.
On exit, SUMSQ is overwritten with smsq , the basic sum of
squares from which scl has been factored out.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine dlasv2 (double precision F, double precision G, double precision H, double precision SSMIN, double precision SSMAX, double precision SNR, double precision CSR, double precision SNL, double precision CSL)

DLASV2 computes the singular value decomposition of a 2-by-2 triangular matrix.

Purpose:

DLASV2 computes the singular value decomposition of a 2-by-2
triangular matrix
[F G]
[0 H].
On return, abs(SSMAX) is the larger singular value, abs(SSMIN) is the
smaller singular value, and (CSL,SNL) and (CSR,SNR) are the left and
right singular vectors for abs(SSMAX), giving the decomposition

[CSL SNL] [F G] [CSR -SNR] = [SSMAX 0]
[-SNL CSL] [0 H] [SNR CSR] [0 SSMIN].

Parameters

F

F is DOUBLE PRECISION
The (1,1) element of the 2-by-2 matrix.

G

G is DOUBLE PRECISION
The (1,2) element of the 2-by-2 matrix.

H

H is DOUBLE PRECISION
The (2,2) element of the 2-by-2 matrix.



SSMIN

SSMIN is DOUBLE PRECISION
abs(SSMIN) is the smaller singular value.

SSMAX

SSMAX is DOUBLE PRECISION
abs(SSMAX) is the larger singular value.

SNL

SNL is DOUBLE PRECISION

CSL

CSL is DOUBLE PRECISION
The vector (CSL, SNL) is a unit left singular vector for the singular value abs(SSMAX).

SNR

SNR is DOUBLE PRECISION

CSR

CSR is DOUBLE PRECISION
The vector (CSR, SNR) is a unit right singular vector for the singular value abs(SSMAX).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

Any input parameter may be aliased with any output parameter.

Barring over/underflow and assuming a guard digit in subtraction, all output quantities are correct to within a few units in the last place (ulps).

In IEEE arithmetic, the code works correctly if one matrix element is infinite.

Overflow will not occur unless the largest singular value itself overflows or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.)

Underflow is harmless if underflow is gradual. Otherwise, results may correspond to a matrix modified by perturbations of size near the underflow threshold.

integer function ieeeck (integer ISPEC, real ZERO, real ONE)

IEEECK

Purpose:

IEEECK is called from the ILAENV to verify that Infinity and possibly NaN arithmetic is safe (i.e. will not trap).



Parameters

ISPEC

ISPEC is INTEGER
Specifies whether to test just for inifinity arithmetic
or whether to test for infinity and NaN arithmetic.
= 0: Verify infinity arithmetic only.
= 1: Verify infinity and NaN arithmetic.

ZERO

ZERO is REAL
Must contain the value 0.0
This is passed to prevent the compiler from optimizing
away this code.

ONE

ONE is REAL
Must contain the value 1.0
This is passed to prevent the compiler from optimizing
away this code.

RETURN VALUE: INTEGER
= 0: Arithmetic failed to produce the correct answers
= 1: Arithmetic produced the correct answers

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

integer function iladlc (integer M, integer N, double precision, dimension(lda, *) A, integer LDA)

ILADLC scans a matrix for its last non-zero column.

Purpose:

ILADLC scans A for its last non-zero column.

Parameters

M

M is INTEGER
The number of rows of the matrix A.

N

N is INTEGER
The number of columns of the matrix A.

A

A is DOUBLE PRECISION array, dimension (LDA,N)
The m by n matrix A.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley



Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

integer function iladlr (integer M, integer N, double precision, dimension(lda, *) A, integer LDA)

ILADLR scans a matrix for its last non-zero row.

Purpose:

ILADLR scans A for its last non-zero row.

Parameters

M

M is INTEGER
The number of rows of the matrix A.

N

N is INTEGER
The number of columns of the matrix A.

A

A is DOUBLE PRECISION array, dimension (LDA,N)
The m by n matrix A.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

integer function ilaenv (integer ISPEC, character*(*) NAME, character*(*) OPTS, integer N1, integer N2, integer N3, integer N4)

ILAENV

Purpose:

ILAENV is called from the LAPACK routines to choose problem-dependent
parameters for the local environment. See ISPEC for a description of
the parameters.

ILAENV returns an INTEGER
if ILAENV >= 0: ILAENV returns the value of the parameter specified by ISPEC
if ILAENV < 0: if ILAENV = -k, the k-th argument had an illegal value.

This version provides a set of parameters which should give good,
but not optimal, performance on many of the currently available
computers. Users are encouraged to modify this subroutine to set
the tuning parameters for their particular machine using the option
and problem size information in the arguments.

This routine will not function correctly if it is converted to all
lower case. Converting it to all upper case is allowed.



Parameters

ISPEC

ISPEC is INTEGER
Specifies the parameter to be returned as the value of
ILAENV.
= 1: the optimal blocksize; if this value is 1, an unblocked
algorithm will give the best performance.
= 2: the minimum block size for which the block routine
should be used; if the usable block size is less than
this value, an unblocked routine should be used.
= 3: the crossover point (in a block routine, for N less
than this value, an unblocked routine should be used)
= 4: the number of shifts, used in the nonsymmetric
eigenvalue routines (DEPRECATED)
= 5: the minimum column dimension for blocking to be used;
rectangular blocks must have dimension at least k by m,
where k is given by ILAENV(2,...) and m by ILAENV(5,...)
= 6: the crossover point for the SVD (when reducing an m by n
matrix to bidiagonal form, if max(m,n)/min(m,n) exceeds
this value, a QR factorization is used first to reduce
the matrix to a triangular form.)
= 7: the number of processors
= 8: the crossover point for the multishift QR method
for nonsymmetric eigenvalue problems (DEPRECATED)
= 9: maximum size of the subproblems at the bottom of the
computation tree in the divide-and-conquer algorithm
(used by xGELSD and xGESDD)
=10: ieee NaN arithmetic can be trusted not to trap
=11: infinity arithmetic can be trusted not to trap
12 <= ISPEC <= 16:
xHSEQR or related subroutines,
see IPARMQ for detailed explanation

NAME

NAME is CHARACTER*(*)
The name of the calling subroutine, in either upper case or
lower case.

OPTS

OPTS is CHARACTER*(*)
The character options to the subroutine NAME, concatenated
into a single character string. For example, UPLO = 'U',
TRANS = 'T', and DIAG = 'N' for a triangular routine would
be specified as OPTS = 'UTN'.

N1

N1 is INTEGER

N2

N2 is INTEGER

N3

N3 is INTEGER

N4

N4 is INTEGER
Problem dimensions for the subroutine NAME; these may not all
be required.

Author



Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

November 2019

Further Details:

The following conventions have been used when calling ILAENV from the LAPACK routines:

1) OPTS is a concatenation of all of the character options to subroutine NAME, in the same order that they appear in the argument list for NAME, even if they are not used in determining the value of the parameter specified by ISPEC.
2) The problem dimensions N1, N2, N3, N4 are specified in the order that they appear in the argument list for NAME. N1 is used first, N2 second, and so on, and unused problem dimensions are passed a value of -1.
3) The parameter value returned by ILAENV is checked for validity in the calling subroutine. For example, ILAENV is used to retrieve the optimal blocksize for STRTRI as follows:

```
NB = ILAENV( 1, 'STRTRI', UPLO // DIAG, N, -1, -1, -1 )
IF( NB.LE.1 ) NB = MAX( 1, N )
```

integer function ilaenv2stage (integer ISPEC, character*(*) NAME, character*(*) OPTS, integer N1, integer N2, integer N3, integer N4)

ILAENV2STAGE

Purpose:

ILAENV2STAGE is called from the LAPACK routines to choose problem-dependent parameters for the local environment. See ISPEC for a description of the parameters.
It sets problem and machine dependent parameters useful for *_2STAGE and related subroutines.

ILAENV2STAGE returns an INTEGER
if ILAENV2STAGE >= 0: ILAENV2STAGE returns the value of the parameter specified by ISPEC
if ILAENV2STAGE < 0: if ILAENV2STAGE = -k, the k-th argument had an illegal value.

This version provides a set of parameters which should give good, but not optimal, performance on many of the currently available computers for the 2-stage solvers. Users are encouraged to modify this subroutine to set the tuning parameters for their particular machine using the option and problem size information in the arguments.

This routine will not function correctly if it is converted to all lower case. Converting it to all upper case is allowed.

Parameters

ISPEC

ISPEC is INTEGER
Specifies the parameter to be returned as the value of ILAENV2STAGE.



= 1: the optimal blocksize nb for the reduction to BAND

= 2: the optimal blocksize ib for the eigenvectors
singular vectors update routine

= 3: The length of the array that store the Housholder
representation for the second stage
Band to Tridiagonal or Bidiagonal

= 4: The workspace needed for the routine in input.

= 5: For future release.

NAME

NAME is CHARACTER*(*)
The name of the calling subroutine, in either upper case or
lower case.

OPTS

OPTS is CHARACTER*(*)
The character options to the subroutine NAME, concatenated
into a single character string. For example, UPLO = 'U',
TRANS = 'T', and DIAG = 'N' for a triangular routine would
be specified as OPTS = 'UTN'.

N1

N1 is INTEGER

N2

N2 is INTEGER

N3

N3 is INTEGER

N4

N4 is INTEGER
Problem dimensions for the subroutine NAME; these may not all
be required.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Nick R. Papior

Date

July 2017

Further Details:

The following conventions have been used when calling ILAENV2STAGE
from the LAPACK routines:
1) OPTS is a concatenation of all of the character options to
subroutine NAME, in the same order that they appear in the
argument list for NAME, even if they are not used in determining
the value of the parameter specified by ISPEC.
2) The problem dimensions N1, N2, N3, N4 are specified in the order
that they appear in the argument list for NAME. N1 is used
first, N2 second, and so on, and unused problem dimensions are



passed a value of -1.

3) The parameter value returned by ILAENV2STAGE is checked for validity in the calling subroutine.

integer function iparmq (integer ISPEC, character, dimension(*) NAME, character, dimension(*) OPTS, integer N, integer ILO, integer IHI, integer LWORK)

IPARMQ

Purpose:

This program sets problem and machine dependent parameters useful for xHSEQR and related subroutines for eigenvalue problems. It is called whenever IPARMQ is called with 12 <= ISPEC <= 16

Parameters

ISPEC

ISPEC is INTEGER

ISPEC specifies which tunable parameter IPARMQ should return.

ISPEC=12: (INMIN) Matrices of order nmin or less are sent directly to xLAHQR, the implicit double shift QR algorithm. NMIN must be at least 11.

ISPEC=13: (INWIN) Size of the deflation window. This is best set greater than or equal to the number of simultaneous shifts NS. Larger matrices benefit from larger deflation windows.

ISPEC=14: (INIBL) Determines when to stop nibbling and invest in an (expensive) multi-shift QR sweep. If the aggressive early deflation subroutine finds LD converged eigenvalues from an order NW deflation window and LD > (NW*NIBBLE)/100, then the next QR sweep is skipped and early deflation is applied immediately to the remaining active diagonal block. Setting IPARMQ(ISPEC=14) = 0 causes TTQRE to skip a multi-shift QR sweep whenever early deflation finds a converged eigenvalue. Setting IPARMQ(ISPEC=14) greater than or equal to 100 prevents TTQRE from skipping a multi-shift QR sweep.

ISPEC=15: (NSHFTS) The number of simultaneous shifts in a multi-shift QR iteration.

ISPEC=16: (IACC22) IPARMQ is set to 0, 1 or 2 with the following meanings.

0: During the multi-shift QR/QZ sweep, blocked eigenvalue reordering, blocked Hessenberg-triangular reduction, reflections and/or rotations are not accumulated when updating the far-from-diagonal matrix entries.

1: During the multi-shift QR/QZ sweep, blocked eigenvalue reordering, blocked



Hessenberg-triangular reduction,
reflections and/or rotations are
accumulated, and matrix-matrix
multiplication is used to update the
far-from-diagonal matrix entries.
2: During the multi-shift QR/QZ sweep,
blocked eigenvalue reordering, blocked
Hessenberg-triangular reduction,
reflections and/or rotations are
accumulated, and 2-by-2 block structure
is exploited during matrix-matrix
multiplies.
(If xTRMM is slower than xGEMM, then
IPARMQ(ISPEC=16)=1 may be more efficient than
IPARMQ(ISPEC=16)=2 despite the greater level of
arithmetic work implied by the latter choice.)

NAME

NAME is CHARACTER string
Name of the calling subroutine

OPTS

OPTS is CHARACTER string
This is a concatenation of the string arguments to
TTQRE.

N

N is INTEGER
N is the order of the Hessenberg matrix H.

ILO

ILO is INTEGER

IHI

IHI is INTEGER
It is assumed that H is already upper triangular
in rows and columns 1:ILO-1 and IHI+1:N.

LWORK

LWORK is INTEGER
The amount of workspace available.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Further Details:

Little is known about how best to choose these parameters.
It is possible to use different values of the parameters
for each of CHSEQR, DHSEQR, SHSEQR and ZHSEQR.

It is probably best to choose different parameters for
different matrices and different parameters at different
times during the iteration, but this has not been



implemented --- yet.

The best choices of most of the parameters depend
in an ill-understood way on the relative execution
rate of xLAQR3 and xLAQR5 and on the nature of each
particular eigenvalue problem. Experiment may be the
only practical way to determine which choices are most
effective.

Following is a list of default values supplied by IPARMQ.
These defaults may be adjusted in order to attain better
performance in any particular computational environment.

IPARMQ(ISPEC=12) The xLAHQR vs xLAQR0 crossover point.
Default: 75. (Must be at least 11.)

IPARMQ(ISPEC=13) Recommended deflation window size.
This depends on ILO, IHI and NS, the
number of simultaneous shifts returned
by IPARMQ(ISPEC=15). The default for
(IHI-ILO+1) <= 500 is NS. The default
for (IHI-ILO+1) > 500 is 3*NS/2.

IPARMQ(ISPEC=14) Nibble crossover point. Default: 14.

IPARMQ(ISPEC=15) Number of simultaneous shifts, NS.
a multi-shift QR iteration.

If IHI-ILO+1 is ...

greater than or equal to ...	...but less than	... the default is
0	30	NS = 2+
30	60	NS = 4+
60	150	NS = 10
150	590	NS = **
590	3000	NS = 64
3000	6000	NS = 128
6000	infinity	NS = 256

(+) By default matrices of this order are
passed to the implicit double shift routine
xLAHQR. See IPARMQ(ISPEC=12) above. These
values of NS are used only in case of a rare
xLAHQR failure.

(**) The asterisks (**) indicate an ad-hoc
function increasing from 10 to 64.

IPARMQ(ISPEC=16) Select structured matrix multiply.
(See ISPEC=16 above for details.)
Default: 3.

logical function lsamen (integer N, character*(*) CA, character*(*) CB)
LSAMEN

Purpose:



LSAMEN tests if the first N letters of CA are the same as the
first N letters of CB, regardless of case.
LSAMEN returns .TRUE. if CA and CB are equivalent except for case
and .FALSE. otherwise. LSAMEN also returns .FALSE. if LEN(CA)
or LEN(CB) is less than N.

Parameters

N

N is INTEGER
The number of characters in CA and CB to be compared.

CA

CA is CHARACTER*(*)

CB

CB is CHARACTER*(*)
CA and CB specify two character strings of length at least N.
Only the first N characters of each string will be accessed.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine scombssq (real, dimension(2) V1, real, dimension(2) V2)

SCOMBSSQ adds two scaled sum of squares quantities

Purpose:

SCOMBSSQ adds two scaled sum of squares quantities, V1 := V1 + V2.
That is,

```
V1_scale**2 * V1_sumsq := V1_scale**2 * V1_sumsq
                        + V2_scale**2 * V2_sumsq
```

Parameters

V1

V1 is REAL array, dimension (2).
The first scaled sum.
V1(1) = V1_scale, V1(2) = V1_sumsq.

V2

V2 is REAL array, dimension (2).
The second scaled sum.
V2(1) = V2_scale, V2(2) = V2_sumsq.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

November 2018



logical function sisnan (real, intent(in) SIN)

SISNAN tests input for NaN.

Purpose:

SISNAN returns .TRUE. if its argument is NaN, and .FALSE. otherwise. To be replaced by the Fortran 2003 intrinsic in the future.

Parameters

SIN

SIN is REAL
Input to test for NaN.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

subroutine slabad (real SMALL, real LARGE)

SLABAD

Purpose:

SLABAD takes as input the values computed by SLAMCH for underflow and overflow, and returns the square root of each of these values if the log of LARGE is sufficiently large. This subroutine is intended to identify machines with a large exponent range, such as the Crays, and redefine the underflow and overflow limits to be the square roots of the values computed by SLAMCH. This subroutine is needed because SLAMCH does not compensate for poor arithmetic in the upper half of the exponent range, as is found on a Cray.

Parameters

SMALL

SMALL is REAL
On entry, the underflow threshold as computed by SLAMCH.
On exit, if LOG10(LARGE) is sufficiently large, the square root of SMALL, otherwise unchanged.

LARGE

LARGE is REAL
On entry, the overflow threshold as computed by SLAMCH.
On exit, if LOG10(LARGE) is sufficiently large, the square root of LARGE, otherwise unchanged.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016



subroutine slacpy (character UPLO, integer M, integer N, real, dimension(lda, *) A, integer LDA, real, dimension(ldb, *) B, integer LDB)

SLACPY copies all or part of one two-dimensional array to another.

Purpose:

SLACPY copies all or part of a two-dimensional matrix A to another matrix B.

Parameters

UPLO

UPLO is CHARACTER*1
Specifies the part of the matrix A to be copied to B.
= 'U': Upper triangular part
= 'L': Lower triangular part
Otherwise: All of the matrix A

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

A

A is REAL array, dimension (LDA,N)
The m by n matrix A. If UPLO = 'U', only the upper triangle or trapezoid is accessed; if UPLO = 'L', only the lower triangle or trapezoid is accessed.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

B

B is REAL array, dimension (LDB,N)
On exit, B = A in the locations specified by UPLO.

LDB

LDB is INTEGER
The leading dimension of the array B. LDB >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine slae2 (real A, real B, real C, real RT1, real RT2)

SLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix.

Purpose:

SLAE2 computes the eigenvalues of a 2-by-2 symmetric matrix
[A B]
[B C].
On return, RT1 is the eigenvalue of larger absolute value, and RT2



is the eigenvalue of smaller absolute value.

Parameters

A

A is REAL
The (1,1) element of the 2-by-2 matrix.

B

B is REAL
The (1,2) and (2,1) elements of the 2-by-2 matrix.

C

C is REAL
The (2,2) element of the 2-by-2 matrix.

RT1

RT1 is REAL
The eigenvalue of larger absolute value.

RT2

RT2 is REAL
The eigenvalue of smaller absolute value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

RT1 is accurate to a few ulps barring over/underflow.

RT2 may be inaccurate if there is massive cancellation in the
determinant A*C-B*B; higher precision or correctly rounded or
correctly truncated arithmetic would be needed to compute RT2
accurately in all cases.

Overflow is possible only if RT1 is within a factor of 5 of overflow.
Underflow is harmless if the input data is 0 or exceeds
underflow_threshold / macheps.

subroutine slaebz (integer IJOB, integer NITMAX, integer N, integer MMAX, integer MINP, integer NBMIN, real ABSTOL, real RELTOL, real PIVMIN, real, dimension(*) D, real, dimension(*) E, real, dimension(*) E2, integer, dimension(*) NVAL, real, dimension(mmax, *) AB, real, dimension(*) C, integer MOUT, integer, dimension(mmax, *) NAB, real, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

SLAEBZ computes the number of eigenvalues of a real symmetric tridiagonal matrix which are less than or equal to a given value, and performs other tasks required by the routine sstebz.

Purpose:

SLAEBZ contains the iteration loops which compute and use the
function N(w), which is the count of eigenvalues of a symmetric
tridiagonal matrix T less than or equal to its argument w. It
performs a choice of two types of loops:

IJOB=1, followed by



IJOB=2: It takes as input a list of intervals and returns a list of
sufficiently small intervals whose union contains the same
eigenvalues as the union of the original intervals.
The input intervals are (AB(j,1),AB(j,2)], j=1,...,MINP.
The output interval (AB(j,1),AB(j,2)] will contain
eigenvalues NAB(j,1)+1,...,NAB(j,2), where 1 <= j <= MOUT.

IJOB=3: It performs a binary search in each input interval
(AB(j,1),AB(j,2)] for a point w(j) such that
N(w(j))=NVAL(j), and uses C(j) as the starting point of
the search. If such a w(j) is found, then on output
AB(j,1)=AB(j,2)=w. If no such w(j) is found, then on output
(AB(j,1),AB(j,2)] will be a small interval containing the
point where N(w) jumps through NVAL(j), unless that point
lies outside the initial interval.

Note that the intervals are in all cases half-open intervals,
i.e., of the form (a,b] , which includes b but not a .

To avoid underflow, the matrix should be scaled so that its largest
element is no greater than overflow**(1/2) * underflow**(1/4)
in absolute value. To assure the most accurate computation
of small eigenvalues, the matrix should be scaled to be
not much smaller than that, either.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal
Matrix", Report CS41, Computer Science Dept., Stanford
University, July 21, 1966

Note: the arguments are, in general, *not* checked for unreasonable
values.

Parameters

IJOB

IJOB is INTEGER
Specifies what is to be done:
= 1: Compute NAB for the initial intervals.
= 2: Perform bisection iteration to find eigenvalues of T.
= 3: Perform bisection iteration to invert N(w), i.e.,
to find a point which has a specified number of
eigenvalues of T to its left.
Other values will cause SLAEBZ to return with INFO=-1.

NITMAX

NITMAX is INTEGER
The maximum number of "levels" of bisection to be
performed, i.e., an interval of width W will not be made
smaller than 2^(-NITMAX) * W. If not all intervals
have converged after NITMAX iterations, then INFO is set
to the number of non-converged intervals.

N

N is INTEGER
The dimension n of the tridiagonal matrix T. It must be at
least 1.

MMAX

MMAX is INTEGER
The maximum number of intervals. If more than MMAX intervals



are generated, then SLAEBZ will quit with INFO=MMAX+1.

MINP

MINP is INTEGER
The initial number of intervals. It may not be greater than MMAX.

NBMIN

NBMIN is INTEGER
The smallest number of intervals that should be processed using a vector loop. If zero, then only the scalar loop will be used.

ABSTOL

ABSTOL is REAL
The minimum (absolute) width of an interval. When an interval is narrower than ABSTOL, or than RELTOL times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. This must be at least zero.

RELTOL

RELTOL is REAL
The minimum relative width of an interval. When an interval is narrower than ABSTOL, or than RELTOL times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. Note: this should always be at least radix*machine epsilon.

PIVMIN

PIVMIN is REAL
The minimum absolute value of a "pivot" in the Sturm sequence loop.
This must be at least max |e(j)**2|*safe_min and at least safe_min, where safe_min is at least the smallest number that can divide one without overflow.

D

D is REAL array, dimension (N)
The diagonal elements of the tridiagonal matrix T.

E

E is REAL array, dimension (N)
The offdiagonal elements of the tridiagonal matrix T in positions 1 through N-1. E(N) is arbitrary.

E2

E2 is REAL array, dimension (N)
The squares of the offdiagonal elements of the tridiagonal matrix T. E2(N) is ignored.

NVAL

NVAL is INTEGER array, dimension (MINP)
If IJOB=1 or 2, not referenced.
If IJOB=3, the desired values of N(w). The elements of NVAL will be reordered to correspond with the intervals in AB.
Thus, NVAL(j) on output will not, in general be the same as NVAL(j) on input, but it will correspond with the interval (AB(j,1),AB(j,2)] on output.

AB



AB is REAL array, dimension (MMAX,2)
The endpoints of the intervals. AB(j,1) is a(j), the left endpoint of the j-th interval, and AB(j,2) is b(j), the right endpoint of the j-th interval. The input intervals will, in general, be modified, split, and reordered by the calculation.

C

C is REAL array, dimension (MMAX)
If IJOB=1, ignored.
If IJOB=2, workspace.
If IJOB=3, then on input C(j) should be initialized to the first search point in the binary search.

MOUT

MOUT is INTEGER
If IJOB=1, the number of eigenvalues in the intervals.
If IJOB=2 or 3, the number of intervals output.
If IJOB=3, MOUT will equal MINP.

NAB

NAB is INTEGER array, dimension (MMAX,2)
If IJOB=1, then on output NAB(i,j) will be set to N(AB(i,j)).
If IJOB=2, then on input, NAB(i,j) should be set. It must satisfy the condition:
N(AB(i,1)) <= NAB(i,1) <= NAB(i,2) <= N(AB(i,2)),
which means that in interval i only eigenvalues NAB(i,1)+1,...,NAB(i,2) will be considered. Usually, NAB(i,j)=N(AB(i,j)), from a previous call to SLAEBZ with IJOB=1.
On output, NAB(i,j) will contain max(na(k),min(nb(k),N(AB(i,j)))), where k is the index of the input interval that the output interval (AB(j,1),AB(j,2)] came from, and na(k) and nb(k) are the the input values of NAB(k,1) and NAB(k,2).
If IJOB=3, then on output, NAB(i,j) contains N(AB(i,j)), unless N(w) > NVAL(i) for all search points w , in which case NAB(i,1) will not be modified, i.e., the output value will be the same as the input value (modulo reorderings -- see NVAL and AB), or unless N(w) < NVAL(i) for all search points w , in which case NAB(i,2) will not be modified. Normally, NAB should be set to some distinctive value(s) before SLAEBZ is called.

WORK

WORK is REAL array, dimension (MMAX)
Workspace.

IWORK

IWORK is INTEGER array, dimension (MMAX)
Workspace.

INFO

INFO is INTEGER
= 0: All intervals converged.
= 1--MMAX: The last INFO intervals did not converge.
= MMAX+1: More than MMAX intervals were generated.

Author

Univ. of Tennessee



Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

This routine is intended to be called only by other LAPACK routines, thus the interface is less user-friendly. It is intended for two purposes:

(a) finding eigenvalues. In this case, SLAEBZ should have one or more initial intervals set up in AB, and SLAEBZ should be called with IJOB=1. This sets up NAB, and also counts the eigenvalues. Intervals with no eigenvalues would usually be thrown out at this point. Also, if not all the eigenvalues in an interval i are desired, NAB(i,1) can be increased or NAB(i,2) decreased. For example, set NAB(i,1)=NAB(i,2)-1 to get the largest eigenvalue. SLAEBZ is then called with IJOB=2 and MMAX no smaller than the value of MOUT returned by the call with IJOB=1. After this (IJOB=2) call, eigenvalues NAB(i,1)+1 through NAB(i,2) are approximately AB(i,1) (or AB(i,2)) to the tolerance specified by ABSTOL and RELTOL.

(b) finding an interval (a',b'] containing eigenvalues w(f),...,w(l). In this case, start with a Gershgorin interval (a,b). Set up AB to contain 2 search intervals, both initially (a,b). One NVAL element should contain f-1 and the other should contain l , while C should contain a and b, resp. NAB(i,1) should be -1 and NAB(i,2) should be N+1, to flag an error if the desired interval does not lie in (a,b). SLAEBZ is then called with IJOB=3. On exit, if w(f-1) < w(f), then one of the intervals -- j -- will have AB(j,1)=AB(j,2) and NAB(j,1)=NAB(j,2)=f-1, while if, to the specified tolerance, w(f-k)=...=w(f+r), k > 0 and r >= 0, then the interval will have N(AB(j,1))=NAB(j,1)=f-k and N(AB(j,2))=NAB(j,2)=f+r. The cases w(l) < w(l+1) and w(l-r)=...=w(l+k) are handled similarly.

subroutine slaev2 (real A, real B, real C, real RT1, real RT2, real CS1, real SN1)

SLAEV2 computes the eigenvalues and eigenvectors of a 2-by-2 symmetric/Hermitian matrix.

Purpose:

SLAEV2 computes the eigendecomposition of a 2-by-2 symmetric matrix
[A B]
[B C].
On return, RT1 is the eigenvalue of larger absolute value, RT2 is the eigenvalue of smaller absolute value, and (CS1,SN1) is the unit right eigenvector for RT1, giving the decomposition

[CS1 SN1] [A B] [CS1 -SN1] = [RT1 0]
[-SN1 CS1] [B C] [SN1 CS1] [0 RT2].

Parameters

A

A is REAL
The (1,1) element of the 2-by-2 matrix.



B

B is REAL
The (1,2) element and the conjugate of the (2,1) element of the 2-by-2 matrix.

C

C is REAL
The (2,2) element of the 2-by-2 matrix.

RT1

RT1 is REAL
The eigenvalue of larger absolute value.

RT2

RT2 is REAL
The eigenvalue of smaller absolute value.

CS1

CS1 is REAL

SN1

SN1 is REAL
The vector (CS1, SN1) is a unit right eigenvector for RT1.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

RT1 is accurate to a few ulps barring over/underflow.

RT2 may be inaccurate if there is massive cancellation in the determinant A*C-B*B; higher precision or correctly rounded or correctly truncated arithmetic would be needed to compute RT2 accurately in all cases.

CS1 and SN1 are accurate to a few ulps barring over/underflow.

Overflow is possible only if RT1 is within a factor of 5 of overflow.
Underflow is harmless if the input data is 0 or exceeds
underflow_threshold / macheps.

subroutine slag2d (integer M, integer N, real, dimension(ldsa, *) SA, integer LDSA, double precision, dimension(lda, *) A, integer LDA, integer INFO)

SLAG2D converts a single precision matrix to a double precision matrix.

Purpose:

SLAG2D converts a SINGLE PRECISION matrix, SA, to a DOUBLE PRECISION matrix, A.

Note that while it is possible to overflow while converting from double to single, it is not possible to overflow when converting from single to double.



This is an auxiliary routine so there is no argument checking.

Parameters

M

M is INTEGER
The number of lines of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

SA

SA is REAL array, dimension (LDSA,N)
On entry, the M-by-N coefficient matrix SA.

LDSA

LDSA is INTEGER
The leading dimension of the array SA. LDSA >= max(1,M).

A

A is DOUBLE PRECISION array, dimension (LDA,N)
On exit, the M-by-N coefficient matrix A.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

INFO

INFO is INTEGER
= 0: successful exit

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine slagts (integer JOB, integer N, real, dimension(*) A, real, dimension(*) B, real, dimension(*) C, real, dimension(*) D, integer, dimension(*) IN, real, dimension(*) Y, real TOL, integer INFO)

SLAGTS solves the system of equations (T-λI)x = y or (T-λI)Tx = y,where T is a general tridiagonal matrix and λ a scalar, using the LU factorization computed by slagtf.

Purpose:

SLAGTS may be used to solve one of the systems of equations

(T - lambda*I)*x = y or (T - lambda*I)**T*x = y,

where T is an n by n tridiagonal matrix, for x, following the factorization of (T - lambda*I) as

(T - lambda*I) = P*L*U ,

by routine SLAGTF. The choice of equation to be solved is controlled by the argument JOB, and in each case there is an option to perturb zero or very small diagonal elements of U, this option being intended for use in applications such as inverse iteration.



Parameters

JOB

JOB is INTEGER
Specifies the job to be performed by SLAGTS as follows:
= 1: The equations (T - lambda*I)x = y are to be solved, but diagonal elements of U are not to be perturbed.
= -1: The equations (T - lambda*I)x = y are to be solved and, if overflow would otherwise occur, the diagonal elements of U are to be perturbed. See argument TOL below.
= 2: The equations (T - lambda*I)**Tx = y are to be solved, but diagonal elements of U are not to be perturbed.
= -2: The equations (T - lambda*I)**Tx = y are to be solved and, if overflow would otherwise occur, the diagonal elements of U are to be perturbed. See argument TOL below.

N

N is INTEGER
The order of the matrix T.

A

A is REAL array, dimension (N)
On entry, A must contain the diagonal elements of U as returned from SLAGTF.

B

B is REAL array, dimension (N-1)
On entry, B must contain the first super-diagonal elements of U as returned from SLAGTF.

C

C is REAL array, dimension (N-1)
On entry, C must contain the sub-diagonal elements of L as returned from SLAGTF.

D

D is REAL array, dimension (N-2)
On entry, D must contain the second super-diagonal elements of U as returned from SLAGTF.

IN

IN is INTEGER array, dimension (N)
On entry, IN must contain details of the matrix P as returned from SLAGTF.

Y

Y is REAL array, dimension (N)
On entry, the right hand side vector y.
On exit, Y is overwritten by the solution vector x.

TOL

TOL is REAL
On entry, with JOB < 0, TOL should be the minimum perturbation to be made to very small diagonal elements of U. TOL should normally be chosen as about eps*norm(U), where eps is the relative machine precision, but if TOL is supplied as non-positive, then it is reset to eps*max(abs(u(i,j))).
If JOB > 0 then TOL is not referenced.



On exit, TOL is changed as described above, only if TOL is non-positive on entry. Otherwise TOL is unchanged.

INFO

INFO is INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: overflow would occur when computing the INFO(th) element of the solution vector x. This can only occur when JOB is supplied as positive and either means that a diagonal element of U is very small, or that the elements of the right-hand side vector y are very large.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

logical function slaisnan (real, intent(in) SIN1, real, intent(in) SIN2)

SLAISNAN tests input for NaN by comparing two arguments for inequality.

Purpose:

This routine is not for general use. It exists solely to avoid over-optimization in SISNAN.

SLAISNAN checks for NaNs by comparing its two arguments for inequality. NaN is the only floating-point value where NaN != NaN returns .TRUE. To check for NaNs, pass the same variable as both arguments.

A compiler must assume that the two arguments are not the same variable, and the test will not be optimized away. Interprocedural or whole-program optimization may delete this test. The ISNAN functions will be replaced by the correct Fortran 03 intrinsic once the intrinsic is widely available.

Parameters

SIN1

SIN1 is REAL

SIN2

SIN2 is REAL
Two numbers to compare for inequality.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017



integer function slaneg (integer N, real, dimension(*) D, real, dimension(*) LLD, real SIGMA, real PIVMIN, integer R)

SLANEG computes the Sturm count.

Purpose:

SLANEG computes the Sturm count, the number of negative pivots encountered while factoring tridiagonal T - sigma I = L D L^T. This implementation works directly on the factors without forming the tridiagonal matrix T. The Sturm count is also the number of eigenvalues of T less than sigma.

This routine is called from SLARRB.

The current routine does not use the PIVMIN parameter but rather requires IEEE-754 propagation of Infinities and NaNs. This routine also has no input range restrictions but does require default exception handling such that x/0 produces Inf when x is non-zero, and Inf/Inf produces NaN. For more information, see:

Marques, Riedy, and Voemel, "Benefits of IEEE-754 Features in Modern Symmetric Tridiagonal Eigensolvers," SIAM Journal on Scientific Computing, v28, n5, 2006. DOI 10.1137/050641624 (Tech report version in LAWN 172 with the same title.)

Parameters

N

N is INTEGER
The order of the matrix.

D

D is REAL array, dimension (N)
The N diagonal elements of the diagonal matrix D.

LLD

LLD is REAL array, dimension (N-1)
The (N-1) elements L(i)*L(i)*D(i).

SIGMA

SIGMA is REAL
Shift amount in T - sigma I = L D L^T.

PIVMIN

PIVMIN is REAL
The minimum pivot in the Sturm sequence. May be used when zero pivots are encountered on non-IEEE-754 architectures.

R

R is INTEGER
The twist index for the twisted factorization that is used for the negcount.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date



December 2016

Contributors:

Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA
Jason Riedy, University of California, Berkeley, USA

real function slanst (character NORM, integer N, real, dimension(*) D, real, dimension(*) E)

SLANST returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix.

Purpose:

SLANST returns the value of the one norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real symmetric tridiagonal matrix A.

Returns

SLANST

SLANST = (max(abs(A(i,j))), NORM = 'M' or 'm'
(
(norm1(A), NORM = '1', 'O' or 'o'
(
(normI(A), NORM = 'I' or 'i'
(
(normF(A), NORM = 'F', 'f', 'E' or 'e'

where norm1 denotes the one norm of a matrix (maximum column sum), normI denotes the infinity norm of a matrix (maximum row sum) and normF denotes the Frobenius norm of a matrix (square root of sum of squares). Note that max(abs(A(i,j))) is not a consistent matrix norm.

Parameters

NORM

NORM is CHARACTER*1
Specifies the value to be returned in SLANST as described above.

N

N is INTEGER
The order of the matrix A. N >= 0. When N = 0, SLANST is set to zero.

D

D is REAL array, dimension (N)
The diagonal elements of A.

E

E is REAL array, dimension (N-1)
The (n-1) sub-diagonal or super-diagonal elements of A.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016



real function slapy2 (real X, real Y)

SLAPY2 returns sqrt(x2+y2).

Purpose:

SLAPY2 returns sqrt(x**2+y**2), taking care not to cause unnecessary overflow.

Parameters

X

X is REAL

Y

Y is REAL
X and Y specify the values x and y.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

real function slapy3 (real X, real Y, real Z)

SLAPY3 returns sqrt(x2+y2+z2).

Purpose:

SLAPY3 returns sqrt(x**2+y**2+z**2), taking care not to cause unnecessary overflow.

Parameters

X

X is REAL

Y

Y is REAL

Z

Z is REAL
X, Y and Z specify the values x, y and z.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine slarnv (integer IDIST, integer, dimension(4) ISEED, integer N, real, dimension(*) X)

SLARNV returns a vector of random numbers from a uniform or normal distribution.

Purpose:

SLARNV returns a vector of n random real numbers from a uniform or normal distribution.

Parameters



IDIST

IDIST is INTEGER
Specifies the distribution of the random numbers:
= 1: uniform (0,1)
= 2: uniform (-1,1)
= 3: normal (0,1)

ISEED

ISEED is INTEGER array, dimension (4)
On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and ISEED(4) must be odd.
On exit, the seed is updated.

N

N is INTEGER
The number of random numbers to be generated.

X

X is REAL array, dimension (N)
The generated random numbers.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

This routine calls the auxiliary routine SLARUV to generate random real numbers from a uniform (0,1) distribution, in batches of up to 128 using vectorisable code. The Box-Muller method is used to transform numbers from a uniform to a normal distribution.

subroutine slarra (integer N, real, dimension(*) D, real, dimension(*) E, real, dimension(*) E2, real SPLTOL, real TNRM, integer NSPLIT, integer, dimension(*) ISPLIT, integer INFO)

SLARRA computes the splitting points with the specified threshold.

Purpose:

Compute the splitting points with threshold SPLTOL.
SLARRA sets any "small" off-diagonal elements to zero.

Parameters

N

N is INTEGER
The order of the matrix. N > 0.

D

D is REAL array, dimension (N)
On entry, the N diagonal elements of the tridiagonal matrix T.

E

E is REAL array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal



elements of the tridiagonal matrix T; E(N) need not be set.
On exit, the entries E(ISPLIT(I)), 1 <= I <= NSPLIT,
are set to zero, the other entries of E are untouched.

E2

E2 is REAL array, dimension (N)
On entry, the first (N-1) entries contain the SQUARES of the
subdiagonal elements of the tridiagonal matrix T;
E2(N) need not be set.
On exit, the entries E2(ISPLIT(I)),
1 <= I <= NSPLIT, have been set to zero

SPLTOL

SPLTOL is REAL
The threshold for splitting. Two criteria can be used:
SPLTOL<0 : criterion based on absolute off-diagonal value
SPLTOL>0 : criterion that preserves relative accuracy

TNRM

TNRM is REAL
The norm of the matrix.

NSPLIT

NSPLIT is INTEGER
The number of blocks T splits into. 1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into blocks.
The first block consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.

INFO

INFO is INTEGER
= 0: successful exit

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slarrb (integer N, real, dimension(*) D, real, dimension(*) LLD, integer IFIRST, integer ILAST, real RTOL1, real RTOL2, integer OFFSET, real, dimension(*) W, real, dimension(*) WGAP, real, dimension(*) WERR, real, dimension(*) WORK, integer, dimension(*) IWORK, real PIVMIN, real SPDIAM, integer TWIST, integer INFO)

SLARRB provides limited bisection to locate eigenvalues for more accuracy.

Purpose:



Given the relatively robust representation(RRR) L D L^T, SLARRB
does "limited" bisection to refine the eigenvalues of L D L^T,
W(IFIRST-OFFSET) through W(ILAST-OFFSET), to more accuracy. Initial
guesses for these eigenvalues are input in W, the corresponding estimate
of the error in these guesses and their gaps are input in WERR
and WGAP, respectively. During bisection, intervals
[left, right] are maintained by storing their mid-points and
semi-widths in the arrays W and WERR respectively.

Parameters

N

N is INTEGER
The order of the matrix.

D

D is REAL array, dimension (N)
The N diagonal elements of the diagonal matrix D.

LLD

LLD is REAL array, dimension (N-1)
The (N-1) elements L(i)*L(i)*D(i).

IFIRST

IFIRST is INTEGER
The index of the first eigenvalue to be computed.

ILAST

ILAST is INTEGER
The index of the last eigenvalue to be computed.

RTOL1

RTOL1 is REAL

RTOL2

RTOL2 is REAL
Tolerance for the convergence of the bisection intervals.
An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < MAX(RTOL1*GAP, RTOL2*MAX(|LEFT|,|RIGHT|))
where GAP is the (estimated) distance to the nearest
eigenvalue.

OFFSET

OFFSET is INTEGER
Offset for the arrays W, WGAP and WERR, i.e., the IFIRST-OFFSET
through ILAST-OFFSET elements of these arrays are to be used.

W

W is REAL array, dimension (N)
On input, W(IFIRST-OFFSET) through W(ILAST-OFFSET) are
estimates of the eigenvalues of L D L^T indexed IFIRST through
ILAST.
On output, these estimates are refined.

WGAP

WGAP is REAL array, dimension (N-1)
On input, the (estimated) gaps between consecutive
eigenvalues of L D L^T, i.e., WGAP(I-OFFSET) is the gap between
eigenvalues I and I+1. Note that if IFIRST = ILAST
then WGAP(IFIRST-OFFSET) must be set to ZERO.
On output, these gaps are refined.



WERR

WERR is REAL array, dimension (N)
On input, WERR(IFIRST-OFFSET) through WERR(ILAST-OFFSET) are
the errors in the estimates of the corresponding elements in W.
On output, these errors are refined.

WORK

WORK is REAL array, dimension (2*N)
Workspace.

IWORK

IWORK is INTEGER array, dimension (2*N)
Workspace.

PIVMIN

PIVMIN is REAL
The minimum pivot in the Sturm sequence.

SPDIAM

SPDIAM is REAL
The spectral diameter of the matrix.

TWIST

TWIST is INTEGER
The twist index for the twisted factorization that is used
for the negcount.
TWIST = N: Compute negcount from L D L^T - LAMBDA I = L+ D+ L+^T
TWIST = 1: Compute negcount from L D L^T - LAMBDA I = U- D- U-^T
TWIST = R: Compute negcount from L D L^T - LAMBDA I = N(r) D(r) N(r)

INFO

INFO is INTEGER
Error flag.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slarrc (character JOBT, integer N, real VL, real VU, real, dimension(*) D, real, dimension(*) E, real PIVMIN, integer EIGCNT, integer LCNT, integer RCNT, integer INFO)

SLARRC computes the number of eigenvalues of the symmetric tridiagonal matrix.

Purpose:

Find the number of eigenvalues of the symmetric tridiagonal matrix T
that are in the interval (VL,VU] if JOBT = 'T', and of L D L^T
if JOBT = 'L'.

Parameters



JOBT

JOBT is CHARACTER*1
= 'T': Compute Sturm count for matrix T.
= 'L': Compute Sturm count for matrix L D L^T.

N

N is INTEGER
The order of the matrix. N > 0.

VL

VL is REAL
The lower bound for the eigenvalues.

VU

VU is REAL
The upper bound for the eigenvalues.

D

D is REAL array, dimension (N)
JOBT = 'T': The N diagonal elements of the tridiagonal matrix T.
JOBT = 'L': The N diagonal elements of the diagonal matrix D.

E

E is REAL array, dimension (N)
JOBT = 'T': The N-1 offdiagonal elements of the matrix T.
JOBT = 'L': The N-1 offdiagonal elements of the matrix L.

PIVMIN

PIVMIN is REAL
The minimum pivot in the Sturm sequence for T.

EIGCNT

EIGCNT is INTEGER
The number of eigenvalues of the symmetric tridiagonal matrix T
that are in the interval (VL,VU]

LCNT

LCNT is INTEGER

RCNT

RCNT is INTEGER
The left and right negcounts of the interval.

INFO

INFO is INTEGER

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA



subroutine slarrd (character RANGE, character ORDER, integer N, real VL, real VU, integer IL, integer IU, real, dimension(*) GERS, real RELTOL, real, dimension(*) D, real, dimension(*) E, real, dimension(*) E2, real PIVMIN, integer NSPLIT, integer, dimension(*) ISPLIT, integer M, real, dimension(*) W, real, dimension(*) WERR, real WL, real WU, integer, dimension(*) IBLOCK, integer, dimension(*) INDEXW, real, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

SLARRD computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

Purpose:

SLARRD computes the eigenvalues of a symmetric tridiagonal matrix T to suitable accuracy. This is an auxiliary code to be called from SSTEMR.
The user may ask for all eigenvalues, all eigenvalues in the half-open interval (VL, VU], or the IL-th through IU-th eigenvalues.

To avoid overflow, the matrix must be scaled so that its largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal Matrix", Report CS41, Computer Science Dept., Stanford University, July 21, 1966.

Parameters

RANGE

RANGE is CHARACTER*1
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval (VL, VU] will be found.
= 'I': ("Index") the IL-th through IU-th eigenvalues (of the entire matrix) will be found.

ORDER

ORDER is CHARACTER*1
= 'B': ("By Block") the eigenvalues will be grouped by split-off block (see IBLOCK, ISPLIT) and ordered from smallest to largest within the block.
= 'E': ("Entire matrix") the eigenvalues for the entire matrix will be ordered from smallest to largest.

N

N is INTEGER
The order of the tridiagonal matrix T. N >= 0.

VL

VL is REAL
If RANGE='V', the lower bound of the interval to be searched for eigenvalues. Eigenvalues less than or equal to VL, or greater than VU, will not be returned. VL < VU.
Not referenced if RANGE = 'A' or 'I'.

VU

VU is REAL
If RANGE='V', the upper bound of the interval to be searched for eigenvalues. Eigenvalues less than or equal



to VL, or greater than VU, will not be returned. VL < VU.
Not referenced if RANGE = 'A' or 'I'.

IL

IL is INTEGER
If RANGE='I', the index of the
smallest eigenvalue to be returned.
1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0.
Not referenced if RANGE = 'A' or 'V'.

IU

IU is INTEGER
If RANGE='I', the index of the
largest eigenvalue to be returned.
1 <= IL <= IU <= N, if N > 0; IL = 1 and IU = 0 if N = 0.
Not referenced if RANGE = 'A' or 'V'.

GERS

GERS is REAL array, dimension (2*N)
The N Gerschgorin intervals (the i-th Gerschgorin interval
is (GERS(2*i-1), GERS(2*i)).

RELTOL

RELTOL is REAL
The minimum relative width of an interval. When an interval
is narrower than RELTOL times the larger (in
magnitude) endpoint, then it is considered to be
sufficiently small, i.e., converged. Note: this should
always be at least radix*machine epsilon.

D

D is REAL array, dimension (N)
The n diagonal elements of the tridiagonal matrix T.

E

E is REAL array, dimension (N-1)
The (n-1) off-diagonal elements of the tridiagonal matrix T.

E2

E2 is REAL array, dimension (N-1)
The (n-1) squared off-diagonal elements of the tridiagonal matrix T.

PIVMIN

PIVMIN is REAL
The minimum pivot allowed in the Sturm sequence for T.

NSPLIT

NSPLIT is INTEGER
The number of diagonal blocks in the matrix T.
1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into submatrices.
The first submatrix consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.
(Only the first NSPLIT elements will actually be used, but
since the user cannot know a priori what value NSPLIT will
have, N words must be reserved for ISPLIT.)



M

M is INTEGER
The actual number of eigenvalues found. 0 <= M <= N.
(See also the description of INFO=2,3.)

W

W is REAL array, dimension (N)
On exit, the first M elements of W will contain the
eigenvalue approximations. SLARRD computes an interval
I_j = (a_j, b_j] that includes eigenvalue j. The eigenvalue
approximation is given as the interval midpoint
W(j)= (a_j + b_j)/2. The corresponding error is bounded by
WERR(j) = abs(a_j - b_j)/2

WERR

WERR is REAL array, dimension (N)
The error bound on the corresponding eigenvalue approximation
in W.

WL

WL is REAL

WU

WU is REAL
The interval (WL, WU] contains all the wanted eigenvalues.
If RANGE='V', then WL=VL and WU=VU.
If RANGE='A', then WL and WU are the global Gerschgorin bounds
on the spectrum.
If RANGE='I', then WL and WU are computed by SLAEBZ from the
index range specified.

IBLOCK

IBLOCK is INTEGER array, dimension (N)
At each row/column j where E(j) is zero or small, the
matrix T is considered to split into a block diagonal
matrix. On exit, if INFO = 0, IBLOCK(i) specifies to which
block (from 1 to the number of blocks) the eigenvalue W(i)
belongs. (SLARRD may use the remaining N-M elements as
workspace.)

INDEXW

INDEXW is INTEGER array, dimension (N)
The indices of the eigenvalues within each block (submatrix);
for example, INDEXW(i)= j and IBLOCK(i)=k imply that the
i-th eigenvalue W(i) is the j-th eigenvalue in block k.

WORK

WORK is REAL array, dimension (4*N)

IWORK

IWORK is INTEGER array, dimension (3*N)

INFO

INFO is INTEGER
= 0: successful exit
< 0: if INFO = -i, the i-th argument had an illegal value
> 0: some or all of the eigenvalues failed to converge or
were not computed:
=1 or 3: Bisection failed to converge for some
eigenvalues; these eigenvalues are flagged by a



negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. This is generally caused by unexpectedly inaccurate arithmetic.

=2 or 3: RANGE='I' only: Not all of the eigenvalues IL:IU were found.
Effect: M < IU+1-IL
Cause: non-monotonic arithmetic, causing the Sturm sequence to be non-monotonic.
Cure: recalculate, using RANGE='A', and pick out eigenvalues IL:IU. In some cases, increasing the PARAMETER "FUDGE" may make things work.

= 4: RANGE='I', and the Gershgorin interval initially used was too small. No eigenvalues were computed.
Probable cause: your machine has sloppy floating-point arithmetic.
Cure: Increase the PARAMETER "FUDGE", recompile, and try again.

Internal Parameters:

FUDGE REAL, default = 2
A "fudge factor" to widen the Gershgorin intervals. Ideally, a value of 1 should work, but on machines with sloppy arithmetic, this needs to be larger. The default for publicly released versions should be large enough to handle the worst machine around. Note that this has no effect on accuracy of the solution.

Contributors:

W. Kahan, University of California, Berkeley, USA
Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

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subroutine slarre (character RANGE, integer N, real VL, real VU, integer IL, integer IU, real, dimension(*) D, real, dimension(*) E, real, dimension(*) E2, real RTOL1, real RTOL2, real SPLTOL, integer NSPLIT, integer, dimension(*) ISPLIT, integer M, real, dimension(*) W, real, dimension(*) WERR, real, dimension(*) WGAP, integer, dimension(*) IBLOCK, integer, dimension(*) INDEXW, real, dimension(*) GERS, real PIVMIN, real, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

SLARRE given the tridiagonal matrix T, sets small off-diagonal elements to zero and for each unreduced block Ti, finds base representations and eigenvalues.

Purpose:



To find the desired eigenvalues of a given real symmetric
tridiagonal matrix T, SLARRE sets any "small" off-diagonal
elements to zero, and for each unreduced block T_i, it finds
(a) a suitable shift at one end of the block's spectrum,
(b) the base representation, T_i - sigma_i I = L_i D_i L_iˆT, and
(c) eigenvalues of each L_i D_i L_iˆT.
The representations and eigenvalues found are then used by
SSTEMR to compute the eigenvectors of T.
The accuracy varies depending on whether bisection is used to
find a few eigenvalues or the dqds algorithm (subroutine SLASQ2) to
conpute all and then discard any unwanted one.
As an added benefit, SLARRE also outputs the n
Gerschgorin intervals for the matrices L_i D_i L_iˆT.

Parameters

RANGE

RANGE is CHARACTER*1
= 'A': ("All") all eigenvalues will be found.
= 'V': ("Value") all eigenvalues in the half-open interval
(VL, VU] will be found.
= 'I': ("Index") the IL-th through IU-th eigenvalues (of the
entire matrix) will be found.

N

N is INTEGER
The order of the matrix. N > 0.

VL

VL is REAL
If RANGE='V', the lower bound for the eigenvalues.
Eigenvalues less than or equal to VL, or greater than VU,
will not be returned. VL < VU.
If RANGE='I' or ='A', SLARRE computes bounds on the desired
part of the spectrum.

VU

VU is REAL
If RANGE='V', the upper bound for the eigenvalues.
Eigenvalues less than or equal to VL, or greater than VU,
will not be returned. VL < VU.
If RANGE='I' or ='A', SLARRE computes bounds on the desired
part of the spectrum.

IL

IL is INTEGER
If RANGE='I', the index of the
smallest eigenvalue to be returned.
1 <= IL <= IU <= N.

IU

IU is INTEGER
If RANGE='I', the index of the
largest eigenvalue to be returned.
1 <= IL <= IU <= N.

D

D is REAL array, dimension (N)
On entry, the N diagonal elements of the tridiagonal
matrix T.
On exit, the N diagonal elements of the diagonal



matrices D_i.

E

E is REAL array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal
elements of the tridiagonal matrix T; E(N) need not be set.
On exit, E contains the subdiagonal elements of the unit
bidiagonal matrices L_i. The entries E(ISPLIT(I)),
1 <= I <= NSPLIT, contain the base points sigma_i on output.

E2

E2 is REAL array, dimension (N)
On entry, the first (N-1) entries contain the SQUARES of the
subdiagonal elements of the tridiagonal matrix T;
E2(N) need not be set.
On exit, the entries E2(ISPLIT(I)),
1 <= I <= NSPLIT, have been set to zero

RTOL1

RTOL1 is REAL

RTOL2

RTOL2 is REAL
Parameters for bisection.
An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < MAX(RTOL1*GAP, RTOL2*MAX(|LEFT|,|RIGHT|))

SPLTOL

SPLTOL is REAL
The threshold for splitting.

NSPLIT

NSPLIT is INTEGER
The number of blocks T splits into. 1 <= NSPLIT <= N.

ISPLIT

ISPLIT is INTEGER array, dimension (N)
The splitting points, at which T breaks up into blocks.
The first block consists of rows/columns 1 to ISPLIT(1),
the second of rows/columns ISPLIT(1)+1 through ISPLIT(2),
etc., and the NSPLIT-th consists of rows/columns
ISPLIT(NSPLIT-1)+1 through ISPLIT(NSPLIT)=N.

M

M is INTEGER
The total number of eigenvalues (of all L_i D_i L_iˆT)
found.

W

W is REAL array, dimension (N)
The first M elements contain the eigenvalues. The
eigenvalues of each of the blocks, L_i D_i L_iˆT, are
sorted in ascending order (SLARRE may use the
remaining N-M elements as workspace).

WERR

WERR is REAL array, dimension (N)
The error bound on the corresponding eigenvalue in W.

WGAP

WGAP is REAL array, dimension (N)



The separation from the right neighbor eigenvalue in W.
The gap is only with respect to the eigenvalues of the same block
as each block has its own representation tree.
Exception: at the right end of a block we store the left gap

IBLOCK

IBLOCK is INTEGER array, dimension (N)
The indices of the blocks (submatrices) associated with the
corresponding eigenvalues in W; IBLOCK(i)=1 if eigenvalue
W(i) belongs to the first block from the top, =2 if W(i)
belongs to the second block, etc.

INDEXW

INDEXW is INTEGER array, dimension (N)
The indices of the eigenvalues within each block (submatrix);
for example, INDEXW(i)= 10 and IBLOCK(i)=2 imply that the
i-th eigenvalue W(i) is the 10-th eigenvalue in block 2

GERS

GERS is REAL array, dimension (2*N)
The N Gerschgorin intervals (the i-th Gerschgorin interval
is (GERS(2*i-1), GERS(2*i)).

PIVMIN

PIVMIN is REAL
The minimum pivot in the Sturm sequence for T.

WORK

WORK is REAL array, dimension (6*N)
Workspace.

IWORK

IWORK is INTEGER array, dimension (5*N)
Workspace.

INFO

INFO is INTEGER
= 0: successful exit
> 0: A problem occurred in SLARRE.
< 0: One of the called subroutines signaled an internal problem.
Needs inspection of the corresponding parameter IINFO
for further information.

=-1: Problem in SLARRD.
= 2: No base representation could be found in MAXTRY iterations.
Increasing MAXTRY and recompilation might be a remedy.
=-3: Problem in SLARRB when computing the refined root
representation for SLASQ2.
=-4: Problem in SLARRB when preforming bisection on the
desired part of the spectrum.
=-5: Problem in SLASQ2.
=-6: Problem in SLASQ2.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date



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Further Details:

The base representations are required to suffer very little element growth and consequently define all their eigenvalues to high relative accuracy.

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slarrf (integer N, real, dimension(*) D, real, dimension(*) L, real, dimension(*) LD, integer CLSTRT, integer CLEND, real, dimension(*) W, real, dimension(*) WGAP, real, dimension(*) WERR, real SPDIAM, real CLGAPL, real CLGAPR, real PIVMIN, real SIGMA, real, dimension(*) DPLUS, real, dimension(*) LPLUS, real, dimension(*) WORK, integer INFO)

SLARRF finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

Purpose:

Given the initial representation L D L^T and its cluster of close eigenvalues (in a relative measure), W(CLSTRT), W(CLSTRT+1), ... W(CLEND), SLARRF finds a new relatively robust representation L D L^T - SIGMA I = L(+) D(+) L(+)^T such that at least one of the eigenvalues of L(+) D(+) L(+)^T is relatively isolated.

Parameters

N

N is INTEGER
The order of the matrix (subblock, if the matrix split).

D

D is REAL array, dimension (N)
The N diagonal elements of the diagonal matrix D.

L

L is REAL array, dimension (N-1)
The (N-1) subdiagonal elements of the unit bidiagonal matrix L.

LD

LD is REAL array, dimension (N-1)
The (N-1) elements L(i)*D(i).

CLSTRT

CLSTRT is INTEGER
The index of the first eigenvalue in the cluster.

CLEND

CLEND is INTEGER
The index of the last eigenvalue in the cluster.

W

W is REAL array, dimension
dimension is >= (CLEND-CLSTRT+1)
The eigenvalue APPROXIMATIONS of L D L^T in ascending order.
W(CLSTRT) through W(CLEND) form the cluster of relatively



close eigenalues.

WGAP

WGAP is REAL array, dimension
dimension is >= (CLEND-CLSTRT+1)
The separation from the right neighbor eigenvalue in W.

WERR

WERR is REAL array, dimension
dimension is >= (CLEND-CLSTRT+1)
WERR contain the semiwidth of the uncertainty
interval of the corresponding eigenvalue APPROXIMATION in W

SPDIAM

SPDIAM is REAL
estimate of the spectral diameter obtained from the
Gerschgorin intervals

CLGAPL

CLGAPL is REAL

CLGAPR

CLGAPR is REAL
absolute gap on each end of the cluster.
Set by the calling routine to protect against shifts too close
to eigenvalues outside the cluster.

PIVMIN

PIVMIN is REAL
The minimum pivot allowed in the Sturm sequence.

SIGMA

SIGMA is REAL
The shift used to form L(+) D(+) L(+)^T.

DPLUS

DPLUS is REAL array, dimension (N)
The N diagonal elements of the diagonal matrix D(+).

LPLUS

LPLUS is REAL array, dimension (N-1)
The first (N-1) elements of LPLUS contain the subdiagonal
elements of the unit bidiagonal matrix L(+).

WORK

WORK is REAL array, dimension (2*N)
Workspace.

INFO

INFO is INTEGER
Signals processing OK (=0) or failure (=1)

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slarrj (integer N, real, dimension(*) D, real, dimension(*) E2, integer IFIRST, integer ILAST, real RTOL, integer OFFSET, real, dimension(*) W, real, dimension(*) WERR, real, dimension(*) WORK, integer, dimension(*) IWORK, real PIVMIN, real SPDIAM, integer INFO)

SLARRJ performs refinement of the initial estimates of the eigenvalues of the matrix T.

Purpose:

Given the initial eigenvalue approximations of T, SLARRJ
does bisection to refine the eigenvalues of T,
W(IFIRST-OFFSET) through W(ILAST-OFFSET), to more accuracy. Initial
guesses for these eigenvalues are input in W, the corresponding estimate
of the error in these guesses in WERR. During bisection, intervals
[left, right] are maintained by storing their mid-points and
semi-widths in the arrays W and WERR respectively.

Parameters

N

N is INTEGER
The order of the matrix.

D

D is REAL array, dimension (N)
The N diagonal elements of T.

E2

E2 is REAL array, dimension (N-1)
The Squares of the (N-1) subdiagonal elements of T.

IFIRST

IFIRST is INTEGER
The index of the first eigenvalue to be computed.

ILAST

ILAST is INTEGER
The index of the last eigenvalue to be computed.

RTOL

RTOL is REAL
Tolerance for the convergence of the bisection intervals.
An interval [LEFT,RIGHT] has converged if
RIGHT-LEFT < RTOL*MAX(|LEFT|,|RIGHT|).

OFFSET

OFFSET is INTEGER
Offset for the arrays W and WERR, i.e., the IFIRST-OFFSET
through ILAST-OFFSET elements of these arrays are to be used.

W

W is REAL array, dimension (N)
On input, W(IFIRST-OFFSET) through W(ILAST-OFFSET) are
estimates of the eigenvalues of L D L^T indexed IFIRST through
ILAST.
On output, these estimates are refined.



WERR

WERR is REAL array, dimension (N)
On input, WERR(IFIRST-OFFSET) through WERR(ILAST-OFFSET) are
the errors in the estimates of the corresponding elements in W.
On output, these errors are refined.

WORK

WORK is REAL array, dimension (2*N)
Workspace.

IWORK

IWORK is INTEGER array, dimension (2*N)
Workspace.

PIVMIN

PIVMIN is REAL
The minimum pivot in the Sturm sequence for T.

SPDIAM

SPDIAM is REAL
The spectral diameter of T.

INFO

INFO is INTEGER
Error flag.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Beresford Parlett, University of California, Berkeley, USA
Jim Demmel, University of California, Berkeley, USA
Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slarrk (integer N, integer IW, real GL, real GU, real, dimension(*) D, real, dimension(*) E2, real PIVMIN, real RELTOL, real W, real WERR, integer INFO)

SLARRK computes one eigenvalue of a symmetric tridiagonal matrix T to suitable accuracy.

Purpose:

SLARRK computes one eigenvalue of a symmetric tridiagonal
matrix T to suitable accuracy. This is an auxiliary code to be
called from SSTEMR.

To avoid overflow, the matrix must be scaled so that its
largest element is no greater than overflow**(1/2) * underflow**(1/4) in absolute value, and for greatest accuracy, it should not be much smaller than that.

See W. Kahan "Accurate Eigenvalues of a Symmetric Tridiagonal
Matrix", Report CS41, Computer Science Dept., Stanford
University, July 21, 1966.

Parameters



N

N is INTEGER
The order of the tridiagonal matrix T. N >= 0.

IW

IW is INTEGER
The index of the eigenvalues to be returned.

GL

GL is REAL

GU

GU is REAL
An upper and a lower bound on the eigenvalue.

D

D is REAL array, dimension (N)
The n diagonal elements of the tridiagonal matrix T.

E2

E2 is REAL array, dimension (N-1)
The (n-1) squared off-diagonal elements of the tridiagonal matrix T.

PIVMIN

PIVMIN is REAL
The minimum pivot allowed in the Sturm sequence for T.

RELTOL

RELTOL is REAL
The minimum relative width of an interval. When an interval
is narrower than RELTOL times the larger (in
magnitude) endpoint, then it is considered to be
sufficiently small, i.e., converged. Note: this should
always be at least radix*machine epsilon.

W

W is REAL

WERR

WERR is REAL
The error bound on the corresponding eigenvalue approximation
in W.

INFO

INFO is INTEGER
= 0: Eigenvalue converged
= -1: Eigenvalue did NOT converge

Internal Parameters:

FUDGE REAL , default = 2
A "fudge factor" to widen the Gershgorin intervals.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date



June 2017

subroutine slarrr (integer N, real, dimension(*) D, real, dimension(*) E, integer INFO)

SLARRR performs tests to decide whether the symmetric tridiagonal matrix T warrants expensive computations which guarantee high relative accuracy in the eigenvalues.

Purpose:

Perform tests to decide whether the symmetric tridiagonal matrix T
warrants expensive computations which guarantee high relative accuracy
in the eigenvalues.

Parameters

N

N is INTEGER
The order of the matrix. N > 0.

D

D is REAL array, dimension (N)
The N diagonal elements of the tridiagonal matrix T.

E

E is REAL array, dimension (N)
On entry, the first (N-1) entries contain the subdiagonal
elements of the tridiagonal matrix T; E(N) is set to ZERO.

INFO

INFO is INTEGER
INFO = 0(default) : the matrix warrants computations preserving
relative accuracy.
INFO = 1 : the matrix warrants computations guaranteeing
only absolute accuracy.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

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Inderjit Dhillon, University of Texas, Austin, USA
Osni Marques, LBNL/NERSC, USA
Christof Voemel, University of California, Berkeley, USA

subroutine slartg (real F, real G, real CS, real SN, real R)

SLARTG generates a plane rotation with real cosine and real sine.

Purpose:

SLARTG generate a plane rotation so that

[CS SN] . [F] = [R] where CS**2 + SN**2 = 1.
[-SN CS] [G] [0]

This is a slower, more accurate version of the BLAS1 routine SROTG,
with the following other differences:
F and G are unchanged on return.



If G=0, then CS=1 and SN=0.
If F=0 and (G .ne. 0), then CS=0 and SN=1 without doing any
floating point operations (saves work in SBDSQR when
there are zeros on the diagonal).

If F exceeds G in magnitude, CS will be positive.

Parameters

F

F is REAL
The first component of vector to be rotated.

G

G is REAL
The second component of vector to be rotated.

CS

CS is REAL
The cosine of the rotation.

SN

SN is REAL
The sine of the rotation.

R

R is REAL
The nonzero component of the rotated vector.

This version has a few statements commented out for thread safety
(machine parameters are computed on each entry). 10 feb 03, SJH.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine slartgp (real F, real G, real CS, real SN, real R)

SLARTGP generates a plane rotation so that the diagonal is nonnegative.

Purpose:

SLARTGP generates a plane rotation so that

[CS SN] . [F] = [R] where CS**2 + SN**2 = 1.
[-SN CS] [G] [0]

This is a slower, more accurate version of the Level 1 BLAS routine SROTG,
with the following other differences:
F and G are unchanged on return.
If G=0, then CS=(+/-)1 and SN=0.
If F=0 and (G .ne. 0), then CS=0 and SN=(+/-)1.

The sign is chosen so that R >= 0.

Parameters

F

F is REAL



The first component of vector to be rotated.

G

G is REAL
The second component of vector to be rotated.

CS

CS is REAL
The cosine of the rotation.

SN

SN is REAL
The sine of the rotation.

R

R is REAL
The nonzero component of the rotated vector.

This version has a few statements commented out for thread safety
(machine parameters are computed on each entry). 10 feb 03, SJH.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

subroutine slaruv (integer, dimension(4) ISEED, integer N, real, dimension(n) X)

SLARUV returns a vector of n random real numbers from a uniform distribution.

Purpose:

SLARUV returns a vector of n random real numbers from a uniform (0,1)
distribution (n <= 128).

This is an auxiliary routine called by SLARNV and CLARNV.

Parameters

ISEED

ISEED is INTEGER array, dimension (4)
On entry, the seed of the random number generator; the array
elements must be between 0 and 4095, and ISEED(4) must be
odd.
On exit, the seed is updated.

N

N is INTEGER
The number of random numbers to be generated. N <= 128.

X

X is REAL array, dimension (N)
The generated random numbers.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver



NAG Ltd.

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Further Details:

This routine uses a multiplicative congruential method with modulus 2**48 and multiplier 33952834046453 (see G.S.Fishman, 'Multiplicative congruential random number generators with modulus 2**b: an exhaustive analysis for b = 32 and a partial analysis for b = 48', Math. Comp. 189, pp 331-344, 1990).

48-bit integers are stored in 4 integer array elements with 12 bits per element. Hence the routine is portable across machines with integers of 32 bits or more.

subroutine slas2 (real F, real G, real H, real SSMIN, real SSMAX)

SLAS2 computes singular values of a 2-by-2 triangular matrix.

Purpose:

SLAS2 computes the singular values of the 2-by-2 matrix
[F G]
[0 H].
On return, SSMIN is the smaller singular value and SSMAX is the larger singular value.

Parameters

F

F is REAL
The (1,1) element of the 2-by-2 matrix.

G

G is REAL
The (1,2) element of the 2-by-2 matrix.

H

H is REAL
The (2,2) element of the 2-by-2 matrix.

SSMIN

SSMIN is REAL
The smaller singular value.

SSMAX

SSMAX is REAL
The larger singular value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

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Further Details:

Barring over/underflow, all output quantities are correct to within



a few units in the last place (ulps), even in the absence of a guard digit in addition/subtraction.

In IEEE arithmetic, the code works correctly if one matrix element is infinite.

Overflow will not occur unless the largest singular value itself overflows, or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.)

Underflow is harmless if underflow is gradual. Otherwise, results may correspond to a matrix modified by perturbations of size near the underflow threshold.

subroutine slascl (character TYPE, integer KL, integer KU, real CFROM, real CTO, integer M, integer N, real, dimension(lda, *) A, integer LDA, integer INFO)

SLASCL multiplies a general rectangular matrix by a real scalar defined as cto/cfrom.

Purpose:

SLASCL multiplies the M by N real matrix A by the real scalar CTO/CFROM. This is done without over/underflow as long as the final result CTO*A(I,J)/CFROM does not over/underflow. TYPE specifies that A may be full, upper triangular, lower triangular, upper Hessenberg, or banded.

Parameters

TYPE

TYPE is CHARACTER*1
TYPE indices the storage type of the input matrix.
= 'G': A is a full matrix.
= 'L': A is a lower triangular matrix.
= 'U': A is an upper triangular matrix.
= 'H': A is an upper Hessenberg matrix.
= 'B': A is a symmetric band matrix with lower bandwidth KL and upper bandwidth KU and with the only the lower half stored.
= 'Q': A is a symmetric band matrix with lower bandwidth KL and upper bandwidth KU and with the only the upper half stored.
= 'Z': A is a band matrix with lower bandwidth KL and upper bandwidth KU. See SGBTRF for storage details.

KL

KL is INTEGER
The lower bandwidth of A. Referenced only if TYPE = 'B', 'Q' or 'Z'.

KU

KU is INTEGER
The upper bandwidth of A. Referenced only if TYPE = 'B', 'Q' or 'Z'.

CFROM

CFROM is REAL

CTO

CTO is REAL



The matrix A is multiplied by CTO/CFROM. A(I,J) is computed without over/underflow if the final result CTO*A(I,J)/CFROM can be represented without over/underflow. CFROM must be nonzero.

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

A

A is REAL array, dimension (LDA,N)
The matrix to be multiplied by CTO/CFROM. See TYPE for the storage type.

LDA

LDA is INTEGER
The leading dimension of the array A.
If TYPE = 'G', 'L', 'U', 'H', LDA >= max(1,M);
TYPE = 'B', LDA >= KL+1;
TYPE = 'Q', LDA >= KU+1;
TYPE = 'Z', LDA >= 2*KL+KU+1.

INFO

INFO is INTEGER
0 - successful exit
<0 - if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine slasd0 (integer N, integer SQRE, real, dimension(*) D, real, dimension(*) E, real, dimension(ldu, *) U, integer LDU, real, dimension(ldvt, *) VT, integer LDVT, integer SMLSIZ, integer, dimension(*) IWORK, real, dimension(*) WORK, integer INFO)

SLASD0 computes the singular values of a real upper bidiagonal n-by-m matrix B with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

Using a divide and conquer approach, SLASD0 computes the singular value decomposition (SVD) of a real upper bidiagonal N-by-M matrix B with diagonal D and offdiagonal E, where M = N + SQRE. The algorithm computes orthogonal matrices U and VT such that B = U * S * VT. The singular values S are overwritten on D.

A related subroutine, SLASDA, computes only the singular values, and optionally, the singular vectors in compact form.

Parameters

N

N is INTEGER
On entry, the row dimension of the upper bidiagonal matrix.



This is also the dimension of the main diagonal array D.

SQRE

SQRE is INTEGER
Specifies the column dimension of the bidiagonal matrix.
= 0: The bidiagonal matrix has column dimension M = N;
= 1: The bidiagonal matrix has column dimension M = N+1;

D

D is REAL array, dimension (N)
On entry D contains the main diagonal of the bidiagonal matrix.
On exit D, if INFO = 0, contains its singular values.

E

E is REAL array, dimension (M-1)
Contains the subdiagonal entries of the bidiagonal matrix.
On exit, E has been destroyed.

U

U is REAL array, dimension (LDU, N)
On exit, U contains the left singular vectors.

LDU

LDU is INTEGER
On entry, leading dimension of U.

VT

VT is REAL array, dimension (LDVT, M)
On exit, VT**T contains the right singular vectors.

LDVT

LDVT is INTEGER
On entry, leading dimension of VT.

SMLSIZ

SMLSIZ is INTEGER
On entry, maximum size of the subproblems at the bottom of the computation tree.

IWORK

IWORK is INTEGER array, dimension (8*N)

WORK

WORK is REAL array, dimension (3*M**2+2*M)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017



Contributors:
Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd1 (integer NL, integer NR, integer SQRE, real, dimension(*) D, real ALPHA, real BETA, real, dimension(ldu, *) U, integer LDU, real, dimension(ldvt, *) VT, integer LDVT, integer, dimension(*) IDXQ, integer, dimension(*) IWORK, real, dimension(*) WORK, integer INFO)

SLASD1 computes the SVD of an upper bidiagonal matrix B of the specified size. Used by sbdsdc.

Purpose:

SLASD1 computes the SVD of an upper bidiagonal N-by-M matrix B,
where N = NL + NR + 1 and M = N + SQRE. SLASD1 is called from SLASD0.

A related subroutine SLASD7 handles the case in which the singular
values (and the singular vectors in factored form) are desired.

SLASD1 computes the SVD as follows:

```
          ( D1(in)    0    0       0 )
B = U(in) * (   Z1**T   a   Z2**T    b ) * VT(in)
          (   0       0   D2(in)   0 )

  = U(out) * ( D(out) 0) * VT(out)
```

where Z**T = (Z1**T a Z2**T b) = u**T VT**T, and u is a vector of dimension M
with ALPHA and BETA in the NL+1 and NL+2 th entries and zeros
elsewhere; and the entry b is empty if SQRE = 0.

The left singular vectors of the original matrix are stored in U, and
the transpose of the right singular vectors are stored in VT, and the
singular values are in D. The algorithm consists of three stages:

The first stage consists of deflating the size of the problem
when there are multiple singular values or when there are zeros in
the Z vector. For each such occurrence the dimension of the
secular equation problem is reduced by one. This stage is
performed by the routine SLASD2.

The second stage consists of calculating the updated
singular values. This is done by finding the square roots of the
roots of the secular equation via the routine SLASD4 (as called
by SLASD3). This routine also calculates the singular vectors of
the current problem.

The final stage consists of computing the updated singular vectors
directly using the updated singular values. The singular vectors
for the current problem are multiplied with the singular vectors
from the overall problem.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE



SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has row dimension N = NL + NR + 1,
and column dimension M = N + SQRE.

D

D is REAL array, dimension (NL+NR+1).
N = NL+NR+1
On entry D(1:NL,1:NL) contains the singular values of the
upper block; and D(NL+2:N) contains the singular values of
the lower block. On exit D(1:N) contains the singular values
of the modified matrix.

ALPHA

ALPHA is REAL
Contains the diagonal element associated with the added row.

BETA

BETA is REAL
Contains the off-diagonal element associated with the added
row.

U

U is REAL array, dimension (LDU,N)
On entry U(1:NL, 1:NL) contains the left singular vectors of
the upper block; U(NL+2:N, NL+2:N) contains the left singular
vectors of the lower block. On exit U contains the left
singular vectors of the bidiagonal matrix.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= max(1, N).

VT

VT is REAL array, dimension (LDVT,M)
where M = N + SQRE.
On entry VT(1:NL+1, 1:NL+1)**T contains the right singular
vectors of the upper block; VT(NL+2:M, NL+2:M)**T contains
the right singular vectors of the lower block. On exit
VT**T contains the right singular vectors of the
bidiagonal matrix.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= max(1, M).

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which will reintegrate the
subproblem just solved back into sorted order, i.e.
D(IDXQ(I = 1, N)) will be in ascending order.

IWORK

IWORK is INTEGER array, dimension (4*N)

WORK

WORK is REAL array, dimension (3*M**2+2*M)

INFO



INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd2 (integer NL, integer NR, integer SQRE, integer K, real, dimension(*) D, real, dimension(*) Z, real ALPHA, real BETA, real, dimension(ldu, *) U, integer LDU, real, dimension(ldvt, *) VT, integer LDVT, real, dimension(*) DSIGMA, real, dimension(ldu2, *) U2, integer LDU2, real, dimension(ldvt2, *) VT2, integer LDVT2, integer, dimension(*) IDXP, integer, dimension(*) IDX, integer, dimension(*) IDXC, integer, dimension(*) IDXQ, integer, dimension(*) COLTYP, integer INFO)

SLASD2 merges the two sets of singular values together into a single sorted set. Used by sbdsdc.

Purpose:

SLASD2 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. There are two ways in which deflation can occur: when two or more singular values are close together or if there is a tiny entry in the Z vector. For each such occurrence the order of the related secular equation problem is reduced by one.

SLASD2 is called from SLASD1.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

K

K is INTEGER
Contains the dimension of the non-deflated matrix,
This is the order of the related secular equation. 1 <= K <=N.

D

D is REAL array, dimension (N)
On entry D contains the singular values of the two submatrices



to be combined. On exit D contains the trailing (N-K) updated singular values (those which were deflated) sorted into increasing order.

Z

Z is REAL array, dimension (N)
On exit Z contains the updating row vector in the secular equation.

ALPHA

ALPHA is REAL
Contains the diagonal element associated with the added row.

BETA

BETA is REAL
Contains the off-diagonal element associated with the added row.

U

U is REAL array, dimension (LDU,N)
On entry U contains the left singular vectors of two submatrices in the two square blocks with corners at (1,1), (NL, NL), and (NL+2, NL+2), (N,N).
On exit U contains the trailing (N-K) updated left singular vectors (those which were deflated) in its last N-K columns.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= N.

VT

VT is REAL array, dimension (LDVT,M)
On entry VT**T contains the right singular vectors of two submatrices in the two square blocks with corners at (1,1), (NL+1, NL+1), and (NL+2, NL+2), (M,M).
On exit VT**T contains the trailing (N-K) updated right singular vectors (those which were deflated) in its last N-K columns.
In case SQRE =1, the last row of VT spans the right null space.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= M.

DSIGMA

DSIGMA is REAL array, dimension (N)
Contains a copy of the diagonal elements (K-1 singular values and one zero) in the secular equation.

U2

U2 is REAL array, dimension (LDU2,N)
Contains a copy of the first K-1 left singular vectors which will be used by SLASD3 in a matrix multiply (SGEMM) to solve for the new left singular vectors. U2 is arranged into four blocks. The first block contains a column with 1 at NL+1 and zero everywhere else; the second block contains non-zero entries only at and above NL; the third contains non-zero entries only below NL+1; and the fourth is dense.

LDU2

LDU2 is INTEGER



The leading dimension of the array U2. LDU2 >= N.

VT2

VT2 is REAL array, dimension (LDVT2,N)
VT2**T contains a copy of the first K right singular vectors
which will be used by SLASD3 in a matrix multiply (SGEMM) to
solve for the new right singular vectors. VT2 is arranged into
three blocks. The first block contains a row that corresponds
to the special 0 diagonal element in SIGMA; the second block
contains non-zeros only at and before NL +1; the third block
contains non-zeros only at and after NL +2.

LDVT2

LDVT2 is INTEGER
The leading dimension of the array VT2. LDVT2 >= M.

IDXP

IDXP is INTEGER array, dimension (N)
This will contain the permutation used to place deflated
values of D at the end of the array. On output IDXP(2:K)
points to the nondeflated D-values and IDXP(K+1:N)
points to the deflated singular values.

IDX

IDX is INTEGER array, dimension (N)
This will contain the permutation used to sort the contents of
D into ascending order.

IDXC

IDXC is INTEGER array, dimension (N)
This will contain the permutation used to arrange the columns
of the deflated U matrix into three groups: the first group
contains non-zero entries only at and above NL, the second
contains non-zero entries only below NL+2, and the third is
dense.

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which separately sorts the two
sub-problems in D into ascending order. Note that entries in
the first hlaf of this permutation must first be moved one
position backward; and entries in the second half
must first have NL+1 added to their values.

COLTYP

COLTYP is INTEGER array, dimension (N)
As workspace, this will contain a label which will indicate
which of the following types a column in the U2 matrix or a
row in the VT2 matrix is:
1 : non-zero in the upper half only
2 : non-zero in the lower half only
3 : dense
4 : deflated

On exit, it is an array of dimension 4, with COLTYP(I) being
the dimension of the I-th type columns.

INFO

INFO is INTEGER
= 0: successful exit.



< 0: if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd3 (integer NL, integer NR, integer SQRE, integer K, real, dimension(*) D, real, dimension(ldq, *) Q, integer LDQ, real, dimension(*) DSIGMA, real, dimension(ldu, *) U, integer LDU, real, dimension(ldu2, *) U2, integer LDU2, real, dimension(ldvt, *) VT, integer LDVT, real, dimension(ldvt2, *) VT2, integer LDVT2, integer, dimension(*) IDXC, integer, dimension(*) CTOT, real, dimension(*) Z, integer INFO)

SLASD3 finds all square roots of the roots of the secular equation, as defined by the values in D and Z, and then updates the singular vectors by matrix multiplication. Used by sbdsdc.

Purpose:

SLASD3 finds all the square roots of the roots of the secular equation, as defined by the values in D and Z. It makes the appropriate calls to SLASD4 and then updates the singular vectors by matrix multiplication.

This code makes very mild assumptions about floating point arithmetic. It will work on machines with a guard digit in add/subtract, or on those binary machines without guard digits which subtract like the Cray XMP, Cray YMP, Cray C 90, or Cray 2. It could conceivably fail on hexadecimal or decimal machines without guard digits, but we know of none.

SLASD3 is called from SLASD1.

Parameters

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

K

K is INTEGER
The size of the secular equation, 1 =< K = < N.

D

D is REAL array, dimension(K)



On exit the square roots of the roots of the secular equation,
in ascending order.

Q

Q is REAL array, dimension (LDQ,K)

LDQ

LDQ is INTEGER
The leading dimension of the array Q. LDQ >= K.

DSIGMA

DSIGMA is REAL array, dimension(K)
The first K elements of this array contain the old roots
of the deflated updating problem. These are the poles
of the secular equation.

U

U is REAL array, dimension (LDU, N)
The last N - K columns of this matrix contain the deflated
left singular vectors.

LDU

LDU is INTEGER
The leading dimension of the array U. LDU >= N.

U2

U2 is REAL array, dimension (LDU2, N)
The first K columns of this matrix contain the non-deflated
left singular vectors for the split problem.

LDU2

LDU2 is INTEGER
The leading dimension of the array U2. LDU2 >= N.

VT

VT is REAL array, dimension (LDVT, M)
The last M - K columns of VT**T contain the deflated
right singular vectors.

LDVT

LDVT is INTEGER
The leading dimension of the array VT. LDVT >= N.

VT2

VT2 is REAL array, dimension (LDVT2, N)
The first K columns of VT2**T contain the non-deflated
right singular vectors for the split problem.

LDVT2

LDVT2 is INTEGER
The leading dimension of the array VT2. LDVT2 >= N.

IDXC

IDXC is INTEGER array, dimension (N)
The permutation used to arrange the columns of U (and rows of
VT) into three groups: the first group contains non-zero
entries only at and above (or before) NL +1; the second
contains non-zero entries only at and below (or after) NL+2;
and the third is dense. The first column of U and the row of
VT are treated separately, however.



The rows of the singular vectors found by SLASD4
must be likewise permuted before the matrix multiplies can
take place.

CTOT

CTOT is INTEGER array, dimension (4)
A count of the total number of the various types of columns
in U (or rows in VT), as described in IDXC. The fourth column
type is any column which has been deflated.

Z

Z is REAL array, dimension (K)
The first K elements of this array contain the components
of the deflation-adjusted updating row vector.

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd4 (integer N, integer I, real, dimension(*) D, real, dimension(*) Z, real, dimension(*) DELTA, real RHO, real SIGMA, real, dimension(*) WORK, integer INFO)

SLASD4 computes the square root of the i-th updated eigenvalue of a positive symmetric rank-one modification to a positive diagonal matrix. Used by sbdsdc.

Purpose:

This subroutine computes the square root of the I-th updated
eigenvalue of a positive symmetric rank-one modification to
a positive diagonal matrix whose entries are given as the squares
of the corresponding entries in the array d, and that

0 <= D(i) < D(j) for i < j

and that RHO > 0. This is arranged by the calling routine, and is
no loss in generality. The rank-one modified system is thus

diag(D) * diag(D) + RHO * Z * Z_transpose.

where we assume the Euclidean norm of Z is 1.

The method consists of approximating the rational functions in the
secular equation by simpler interpolating rational functions.

Parameters

N

N is INTEGER
The length of all arrays.



I

I is INTEGER
The index of the eigenvalue to be computed. 1 <= I <= N.

D

D is REAL array, dimension (N)
The original eigenvalues. It is assumed that they are in order, 0 <= D(I) < D(J) for I < J.

Z

Z is REAL array, dimension (N)
The components of the updating vector.

DELTA

DELTA is REAL array, dimension (N)
If N .ne. 1, DELTA contains (D(j) - sigma_I) in its j-th component. If N = 1, then DELTA(1) = 1. The vector DELTA contains the information necessary to construct the (singular) eigenvectors.

RHO

RHO is REAL
The scalar in the symmetric updating formula.

SIGMA

SIGMA is REAL
The computed sigma_I, the I-th updated eigenvalue.

WORK

WORK is REAL array, dimension (N)
If N .ne. 1, WORK contains (D(j) + sigma_I) in its j-th component. If N = 1, then WORK(1) = 1.

INFO

INFO is INTEGER
= 0: successful exit
> 0: if INFO = 1, the updating process failed.

Internal Parameters:

Logical variable ORGATI (origin-at-i?) is used for distinguishing whether D(i) or D(i+1) is treated as the origin.

ORGATI = .true. origin at i
ORGATI = .false. origin at i+1

Logical variable SWTCH3 (switch-for-3-poles?) is for noting if we are working with THREE poles!

MAXIT is the maximum number of iterations allowed for each eigenvalue.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016



Contributors:
Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

subroutine slasd5 (integer I, real, dimension(2) D, real, dimension(2) Z, real, dimension(2) DELTA, real RHO, real DSIGMA, real, dimension(2) WORK)

SLASD5 computes the square root of the i-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix. Used by sbdsdc.

Purpose:

This subroutine computes the square root of the I-th eigenvalue of a positive symmetric rank-one modification of a 2-by-2 diagonal matrix

diag(D) * diag(D) + RHO * Z * transpose(Z) .

The diagonal entries in the array D are assumed to satisfy

0 <= D(i) < D(j) for i < j .

We also assume RHO > 0 and that the Euclidean norm of the vector Z is one.

Parameters

I

I is INTEGER
The index of the eigenvalue to be computed. I = 1 or I = 2.

D

D is REAL array, dimension (2)
The original eigenvalues. We assume 0 <= D(1) < D(2).

Z

Z is REAL array, dimension (2)
The components of the updating vector.

DELTA

DELTA is REAL array, dimension (2)
Contains (D(j) - sigma_I) in its j-th component.
The vector DELTA contains the information necessary to construct the eigenvectors.

RHO

RHO is REAL
The scalar in the symmetric updating formula.

DSIGMA

DSIGMA is REAL
The computed sigma_I, the I-th updated eigenvalue.

WORK

WORK is REAL array, dimension (2)
WORK contains (D(j) + sigma_I) in its j-th component.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date



December 2016

Contributors:

Ren-Cang Li, Computer Science Division, University of California at Berkeley, USA

subroutine slasd6 (integer ICOMPQ, integer NL, integer NR, integer SQRE, real, dimension(*) D, real, dimension(*) VF, real, dimension(*) VL, real ALPHA, real BETA, integer, dimension(*) IDXQ, integer, dimension(*) PERM, integer GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, real, dimension(ldgnum, *) GIVNUM, integer LDGNUM, real, dimension(ldgnum, *) POLES, real, dimension(*) DIFL, real, dimension(*) DIFR, real, dimension(*) Z, integer K, real C, real S, real, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

SLASD6 computes the SVD of an updated upper bidiagonal matrix obtained by merging two smaller ones by appending a row. Used by sbdsdc.

Purpose:

SLASD6 computes the SVD of an updated upper bidiagonal matrix B obtained by merging two smaller ones by appending a row. This routine is used only for the problem which requires all singular values and optionally singular vector matrices in factored form. B is an N-by-M matrix with N = NL + NR + 1 and M = N + SQRE. A related subroutine, SLASD1, handles the case in which all singular values and singular vectors of the bidiagonal matrix are desired.

SLASD6 computes the SVD as follows:

```
          ( D1(in)    0    0       0 )
B = U(in) * (   Z1**T   a   Z2**T    b ) * VT(in)
          (   0       0   D2(in)   0 )

  = U(out) * ( D(out) 0) * VT(out)
```

where Z**T = (Z1**T a Z2**T b) = u**T VT**T, and u is a vector of dimension M with ALPHA and BETA in the NL+1 and NL+2 th entries and zeros elsewhere; and the entry b is empty if SQRE = 0.

The singular values of B can be computed using D1, D2, the first components of all the right singular vectors of the lower block, and the last components of all the right singular vectors of the upper block. These components are stored and updated in VF and VL, respectively, in SLASD6. Hence U and VT are not explicitly referenced.

The singular values are stored in D. The algorithm consists of two stages:

The first stage consists of deflating the size of the problem when there are multiple singular values or if there is a zero in the Z vector. For each such occurrence the dimension of the secular equation problem is reduced by one. This stage is performed by the routine SLASD7.

The second stage consists of calculating the updated singular values. This is done by finding the roots of the secular equation via the routine SLASD4 (as called by SLASD8). This routine also updates VF and VL and computes the distances between the updated singular values and the old singular values.



SLASD6 is called from SLASDA.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in factored form:
= 0: Compute singular values only.
= 1: Compute singular vectors in factored form as well.

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has row dimension N = NL + NR + 1, and column dimension M = N + SQRE.

D

D is REAL array, dimension (NL+NR+1).
On entry D(1:NL,1:NL) contains the singular values of the upper block, and D(NL+2:N) contains the singular values of the lower block. On exit D(1:N) contains the singular values of the modified matrix.

VF

VF is REAL array, dimension (M)
On entry, VF(1:NL+1) contains the first components of all right singular vectors of the upper block; and VF(NL+2:M) contains the first components of all right singular vectors of the lower block. On exit, VF contains the first components of all right singular vectors of the bidiagonal matrix.

VL

VL is REAL array, dimension (M)
On entry, VL(1:NL+1) contains the last components of all right singular vectors of the upper block; and VL(NL+2:M) contains the last components of all right singular vectors of the lower block. On exit, VL contains the last components of all right singular vectors of the bidiagonal matrix.

ALPHA

ALPHA is REAL
Contains the diagonal element associated with the added row.

BETA

BETA is REAL
Contains the off-diagonal element associated with the added row.

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which will reintegrate the



subproblem just solved back into sorted order, i.e.
D(IDXQ(I = 1, N)) will be in ascending order.

PERM

PERM is INTEGER array, dimension (N)
The permutations (from deflation and sorting) to be applied
to each block. Not referenced if ICOMPQ = 0.

GIVPTR

GIVPTR is INTEGER
The number of Givens rotations which took place in this
subproblem. Not referenced if ICOMPQ = 0.

GIVCOL

GIVCOL is INTEGER array, dimension (LDGCOL, 2)
Each pair of numbers indicates a pair of columns to take place
in a Givens rotation. Not referenced if ICOMPQ = 0.

LDGCOL

LDGCOL is INTEGER
leading dimension of GIVCOL, must be at least N.

GIVNUM

GIVNUM is REAL array, dimension (LDGNUM, 2)
Each number indicates the C or S value to be used in the
corresponding Givens rotation. Not referenced if ICOMPQ = 0.

LDGNUM

LDGNUM is INTEGER
The leading dimension of GIVNUM and POLES, must be at least N.

POLES

POLES is REAL array, dimension (LDGNUM, 2)
On exit, POLES(1,*) is an array containing the new singular
values obtained from solving the secular equation, and
POLES(2,*) is an array containing the poles in the secular
equation. Not referenced if ICOMPQ = 0.

DIFL

DIFL is REAL array, dimension (N)
On exit, DIFL(I) is the distance between I-th updated
(undeflated) singular value and the I-th (undeflated) old
singular value.

DIFR

DIFR is REAL array,
dimension (LDDIFR, 2) if ICOMPQ = 1 and
dimension (K) if ICOMPQ = 0.
On exit, DIFR(I,1) = D(I) - DSIGMA(I+1), DIFR(K,1) is not
defined and will not be referenced.

If ICOMPQ = 1, DIFR(1:K,2) is an array containing the
normalizing factors for the right singular vector matrix.

See SLASD8 for details on DIFL and DIFR.

Z

Z is REAL array, dimension (M)
The first elements of this array contain the components
of the deflation-adjusted updating row vector.



K

K is INTEGER
Contains the dimension of the non-deflated matrix,
This is the order of the related secular equation. 1 <= K <=N.

C

C is REAL
C contains garbage if SQRE =0 and the C-value of a Givens
rotation related to the right null space if SQRE = 1.

S

S is REAL
S contains garbage if SQRE =0 and the S-value of a Givens
rotation related to the right null space if SQRE = 1.

WORK

WORK is REAL array, dimension (4 * M)

IWORK

IWORK is INTEGER array, dimension (3 * N)

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd7 (integer ICOMPQ, integer NL, integer NR, integer SQRE, integer K, real, dimension(*) D, real, dimension(*) Z, real, dimension(*) ZW, real, dimension(*) VF, real, dimension(*) VFW, real, dimension(*) VL, real, dimension(*) VLW, real ALPHA, real BETA, real, dimension(*) DSIGMA, integer, dimension(*) IDX, integer, dimension(*) IDXP, integer, dimension(*) IDXQ, integer, dimension(*) PERM, integer GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, real, dimension(ldgnum, *) GIVNUM, integer LDGNUM, real C, real S, integer INFO)

SLASD7 merges the two sets of singular values together into a single sorted set. Then it tries to deflate the size of the problem. Used by sbdsdc.

Purpose:

SLASD7 merges the two sets of singular values together into a single
sorted set. Then it tries to deflate the size of the problem. There
are two ways in which deflation can occur: when two or more singular
values are close together or if there is a tiny entry in the Z
vector. For each such occurrence the order of the related
secular equation problem is reduced by one.

SLASD7 is called from SLASD6.

Parameters



ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed
in compact form, as follows:
= 0: Compute singular values only.
= 1: Compute singular vectors of upper
bidiagonal matrix in compact form.

NL

NL is INTEGER
The row dimension of the upper block. NL >= 1.

NR

NR is INTEGER
The row dimension of the lower block. NR >= 1.

SQRE

SQRE is INTEGER
= 0: the lower block is an NR-by-NR square matrix.
= 1: the lower block is an NR-by-(NR+1) rectangular matrix.

The bidiagonal matrix has
N = NL + NR + 1 rows and
M = N + SQRE >= N columns.

K

K is INTEGER
Contains the dimension of the non-deflated matrix, this is
the order of the related secular equation. 1 <= K <=N.

D

D is REAL array, dimension (N)
On entry D contains the singular values of the two submatrices
to be combined. On exit D contains the trailing (N-K) updated
singular values (those which were deflated) sorted into
increasing order.

Z

Z is REAL array, dimension (M)
On exit Z contains the updating row vector in the secular
equation.

ZW

ZW is REAL array, dimension (M)
Workspace for Z.

VF

VF is REAL array, dimension (M)
On entry, VF(1:NL+1) contains the first components of all
right singular vectors of the upper block; and VF(NL+2:M)
contains the first components of all right singular vectors
of the lower block. On exit, VF contains the first components
of all right singular vectors of the bidiagonal matrix.

VFW

VFW is REAL array, dimension (M)
Workspace for VF.

VL

VL is REAL array, dimension (M)



On entry, VL(1:NL+1) contains the last components of all right singular vectors of the upper block; and VL(NL+2:M) contains the last components of all right singular vectors of the lower block. On exit, VL contains the last components of all right singular vectors of the bidiagonal matrix.

VLW

VLW is REAL array, dimension (M)
Workspace for VL.

ALPHA

ALPHA is REAL
Contains the diagonal element associated with the added row.

BETA

BETA is REAL
Contains the off-diagonal element associated with the added row.

DSIGMA

DSIGMA is REAL array, dimension (N)
Contains a copy of the diagonal elements (K-1 singular values and one zero) in the secular equation.

IDX

IDX is INTEGER array, dimension (N)
This will contain the permutation used to sort the contents of D into ascending order.

IDXP

IDXP is INTEGER array, dimension (N)
This will contain the permutation used to place deflated values of D at the end of the array. On output IDXP(2:K) points to the nondeflated D-values and IDXP(K+1:N) points to the deflated singular values.

IDXQ

IDXQ is INTEGER array, dimension (N)
This contains the permutation which separately sorts the two sub-problems in D into ascending order. Note that entries in the first half of this permutation must first be moved one position backward; and entries in the second half must first have NL+1 added to their values.

PERM

PERM is INTEGER array, dimension (N)
The permutations (from deflation and sorting) to be applied to each singular block. Not referenced if ICOMPQ = 0.

GIVPTR

GIVPTR is INTEGER
The number of Givens rotations which took place in this subproblem. Not referenced if ICOMPQ = 0.

GIVCOL

GIVCOL is INTEGER array, dimension (LDGCOL, 2)
Each pair of numbers indicates a pair of columns to take place in a Givens rotation. Not referenced if ICOMPQ = 0.

LDGCOL

LDGCOL is INTEGER



The leading dimension of GIVCOL, must be at least N.

GIVNUM

GIVNUM is REAL array, dimension (LDGNUM, 2)
Each number indicates the C or S value to be used in the corresponding Givens rotation. Not referenced if ICOMPQ = 0.

LDGNUM

LDGNUM is INTEGER
The leading dimension of GIVNUM, must be at least N.

C

C is REAL
C contains garbage if SQRE =0 and the C-value of a Givens rotation related to the right null space if SQRE = 1.

S

S is REAL
S contains garbage if SQRE =0 and the S-value of a Givens rotation related to the right null space if SQRE = 1.

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasd8 (integer ICOMPQ, integer K, real, dimension(*) D, real, dimension(*) Z, real, dimension(*) VF, real, dimension(*) VL, real, dimension(*) DIFL, real, dimension(lddifr, *) DIFR, integer LDDIFR, real, dimension(*) DSIGMA, real, dimension(*) WORK, integer INFO)

SLASD8 finds the square roots of the roots of the secular equation, and stores, for each element in D, the distance to its two nearest poles. Used by sbdsdc.

Purpose:

SLASD8 finds the square roots of the roots of the secular equation, as defined by the values in DSIGMA and Z. It makes the appropriate calls to SLASD4, and stores, for each element in D, the distance to its two nearest poles (elements in DSIGMA). It also updates the arrays VF and VL, the first and last components of all the right singular vectors of the original bidiagonal matrix.

SLASD8 is called from SLASD6.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in factored form in the calling routine:



= 0: Compute singular values only.
= 1: Compute singular vectors in factored form as well.

K

K is INTEGER
The number of terms in the rational function to be solved
by SLASD4. K >= 1.

D

D is REAL array, dimension (K)
On output, D contains the updated singular values.

Z

Z is REAL array, dimension (K)
On entry, the first K elements of this array contain the
components of the deflation-adjusted updating row vector.
On exit, Z is updated.

VF

VF is REAL array, dimension (K)
On entry, VF contains information passed through DBEDE8.
On exit, VF contains the first K components of the first
components of all right singular vectors of the bidiagonal
matrix.

VL

VL is REAL array, dimension (K)
On entry, VL contains information passed through DBEDE8.
On exit, VL contains the first K components of the last
components of all right singular vectors of the bidiagonal
matrix.

DIFL

DIFL is REAL array, dimension (K)
On exit, DIFL(I) = D(I) - DSIGMA(I).

DIFR

DIFR is REAL array,
dimension (LDDIFR, 2) if ICOMPQ = 1 and
dimension (K) if ICOMPQ = 0.
On exit, DIFR(I,1) = D(I) - DSIGMA(I+1), DIFR(K,1) is not
defined and will not be referenced.

If ICOMPQ = 1, DIFR(1:K,2) is an array containing the
normalizing factors for the right singular vector matrix.

LDDIFR

LDDIFR is INTEGER
The leading dimension of DIFR, must be at least K.

DSIGMA

DSIGMA is REAL array, dimension (K)
On entry, the first K elements of this array contain the old
roots of the deflated updating problem. These are the poles
of the secular equation.
On exit, the elements of DSIGMA may be very slightly altered
in value.

WORK

WORK is REAL array, dimension (3*K)



INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

June 2017

Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasda (integer ICOMPQ, integer SMLSIZ, integer N, integer SQRE, real, dimension(*) D, real, dimension(*) E, real, dimension(ldu, *) U, integer LDU, real, dimension(ldu, *) VT, integer, dimension(*) K, real, dimension(ldu, *) DIFL, real, dimension(ldu, *) DIFR, real, dimension(ldu, *) Z, real, dimension(ldu, *) POLES, integer, dimension(*) GIVPTR, integer, dimension(ldgcol, *) GIVCOL, integer LDGCOL, integer, dimension(ldgcol, *) PERM, real, dimension(ldu, *) GIVNUM, real, dimension(*) C, real, dimension(*) S, real, dimension(*) WORK, integer, dimension(*) IWORK, integer INFO)

SLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

Using a divide and conquer approach, SLASDA computes the singular value decomposition (SVD) of a real upper bidiagonal N-by-M matrix B with diagonal D and offdiagonal E, where M = N + SQRE. The algorithm computes the singular values in the SVD B = U * S * VT. The orthogonal matrices U and VT are optionally computed in compact form.

A related subroutine, SLASD0, computes the singular values and the singular vectors in explicit form.

Parameters

ICOMPQ

ICOMPQ is INTEGER
Specifies whether singular vectors are to be computed in compact form, as follows
= 0: Compute singular values only.
= 1: Compute singular vectors of upper bidiagonal matrix in compact form.

SMLSIZ

SMLSIZ is INTEGER
The maximum size of the subproblems at the bottom of the computation tree.

N

N is INTEGER
The row dimension of the upper bidiagonal matrix. This is also the dimension of the main diagonal array D.

SQRE



SQRE is INTEGER
Specifies the column dimension of the bidiagonal matrix.
= 0: The bidiagonal matrix has column dimension M = N;
= 1: The bidiagonal matrix has column dimension M = N + 1.

D

D is REAL array, dimension (N)
On entry D contains the main diagonal of the bidiagonal
matrix. On exit D, if INFO = 0, contains its singular values.

E

E is REAL array, dimension (M-1)
Contains the subdiagonal entries of the bidiagonal matrix.
On exit, E has been destroyed.

U

U is REAL array,
dimension (LDU, SMLSIZ) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, U contains the left
singular vector matrices of all subproblems at the bottom
level.

LDU

LDU is INTEGER, LDU = > N.
The leading dimension of arrays U, VT, DIFL, DIFR, POLES,
GIVNUM, and Z.

VT

VT is REAL array,
dimension (LDU, SMLSIZ+1) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, VT**T contains the right
singular vector matrices of all subproblems at the bottom
level.

K

K is INTEGER array, dimension (N)
if ICOMPQ = 1 and dimension 1 if ICOMPQ = 0.
If ICOMPQ = 1, on exit, K(I) is the dimension of the I-th
secular equation on the computation tree.

DIFL

DIFL is REAL array, dimension (LDU, NLVL),
where NLVL = floor(log_2 (N/SMLSIZ))).

DIFR

DIFR is REAL array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1 and
dimension (N) if ICOMPQ = 0.
If ICOMPQ = 1, on exit, DIFL(1:N, I) and DIFR(1:N, 2 * I - 1)
record distances between singular values on the I-th
level and singular values on the (I -1)-th level, and
DIFR(1:N, 2 * I) contains the normalizing factors for
the right singular vector matrix. See SLASD8 for details.

Z

Z is REAL array,
dimension (LDU, NLVL) if ICOMPQ = 1 and
dimension (N) if ICOMPQ = 0.
The first K elements of Z(1, I) contain the components of
the deflation-adjusted updating row vector for subproblems



on the I-th level.

POLES

POLES is REAL array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, POLES(1, 2*I - 1) and
POLES(1, 2*I) contain the new and old singular values
involved in the secular equations on the I-th level.

GIVPTR

GIVPTR is INTEGER array,
dimension (N) if ICOMPQ = 1, and not referenced if
ICOMPQ = 0. If ICOMPQ = 1, on exit, GIVPTR(I) records
the number of Givens rotations performed on the I-th
problem on the computation tree.

GIVCOL

GIVCOL is INTEGER array,
dimension (LDGCOL, 2 * NLVL) if ICOMPQ = 1, and not
referenced if ICOMPQ = 0. If ICOMPQ = 1, on exit, for each I,
GIVCOL(1, 2 *I - 1) and GIVCOL(1, 2 *I) record the locations
of Givens rotations performed on the I-th level on the
computation tree.

LDGCOL

LDGCOL is INTEGER, LDGCOL = > N.
The leading dimension of arrays GIVCOL and PERM.

PERM

PERM is INTEGER array, dimension (LDGCOL, NLVL)
if ICOMPQ = 1, and not referenced
if ICOMPQ = 0. If ICOMPQ = 1, on exit, PERM(1, I) records
permutations done on the I-th level of the computation tree.

GIVNUM

GIVNUM is REAL array,
dimension (LDU, 2 * NLVL) if ICOMPQ = 1, and not
referenced if ICOMPQ = 0. If ICOMPQ = 1, on exit, for each I,
GIVNUM(1, 2 *I - 1) and GIVNUM(1, 2 *I) record the C- and S-
values of Givens rotations performed on the I-th level on
the computation tree.

C

C is REAL array,
dimension (N) if ICOMPQ = 1, and dimension 1 if ICOMPQ = 0.
If ICOMPQ = 1 and the I-th subproblem is not square, on exit,
C(I) contains the C-value of a Givens rotation related to
the right null space of the I-th subproblem.

S

S is REAL array, dimension (N) if
ICOMPQ = 1, and dimension 1 if ICOMPQ = 0. If ICOMPQ = 1
and the I-th subproblem is not square, on exit, S(I)
contains the S-value of a Givens rotation related to
the right null space of the I-th subproblem.

WORK

WORK is REAL array, dimension
(6 * N + (SMLSIZ + 1)*(SMLSIZ + 1)).

IWORK



IWORK is INTEGER array, dimension (7*N).

INFO

INFO is INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if INFO = 1, a singular value did not converge

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasdq (character UPLO, integer SQRE, integer N, integer NCVT, integer NRU, integer NCC, real, dimension(*) D, real, dimension(*) E, real, dimension(ldvt, *) VT, integer LDVT, real, dimension(ldu, *) U, integer LDU, real, dimension(ldc, *) C, integer LDC, real, dimension(*) WORK, integer INFO)

SLASDQ computes the SVD of a real bidiagonal matrix with diagonal d and off-diagonal e. Used by sbdsdc.

Purpose:

SLASDQ computes the singular value decomposition (SVD) of a real
(upper or lower) bidiagonal matrix with diagonal D and offdiagonal
E, accumulating the transformations if desired. Letting B denote
the input bidiagonal matrix, the algorithm computes orthogonal
matrices Q and P such that B = Q * S * P**T (P**T denotes the transpose
of P). The singular values S are overwritten on D.

The input matrix U is changed to U * Q if desired.
The input matrix VT is changed to P**T * VT if desired.
The input matrix C is changed to Q**T * C if desired.

See "Computing Small Singular Values of Bidiagonal Matrices With
Guaranteed High Relative Accuracy," by J. Demmel and W. Kahan,
LAPACK Working Note #3, for a detailed description of the algorithm.

Parameters

UPLO

UPLO is CHARACTER*1
On entry, UPLO specifies whether the input bidiagonal matrix
is upper or lower bidiagonal, and whether it is square are
not.
UPLO = 'U' or 'u' B is upper bidiagonal.
UPLO = 'L' or 'l' B is lower bidiagonal.

SQRE

SQRE is INTEGER
= 0: then the input matrix is N-by-N.
= 1: then the input matrix is N-by-(N+1) if UPLU = 'U' and
(N+1)-by-N if UPLU = 'L'.

The bidiagonal matrix has
N = NL + NR + 1 rows and



M = N + SQRE >= N columns.

N

N is INTEGER
On entry, N specifies the number of rows and columns in the matrix. N must be at least 0.

NCVT

NCVT is INTEGER
On entry, NCVT specifies the number of columns of the matrix VT. NCVT must be at least 0.

NRU

NRU is INTEGER
On entry, NRU specifies the number of rows of the matrix U. NRU must be at least 0.

NCC

NCC is INTEGER
On entry, NCC specifies the number of columns of the matrix C. NCC must be at least 0.

D

D is REAL array, dimension (N)
On entry, D contains the diagonal entries of the bidiagonal matrix whose SVD is desired. On normal exit, D contains the singular values in ascending order.

E

E is REAL array.
dimension is (N-1) if SQRE = 0 and N if SQRE = 1.
On entry, the entries of E contain the offdiagonal entries of the bidiagonal matrix whose SVD is desired. On normal exit, E will contain 0. If the algorithm does not converge, D and E will contain the diagonal and superdiagonal entries of a bidiagonal matrix orthogonally equivalent to the one given as input.

VT

VT is REAL array, dimension (LDVT, NCVT)
On entry, contains a matrix which on exit has been premultiplied by P**T, dimension N-by-NCVT if SQRE = 0 and (N+1)-by-NCVT if SQRE = 1 (not referenced if NCVT=0).

LDVT

LDVT is INTEGER
On entry, LDVT specifies the leading dimension of VT as declared in the calling (sub) program. LDVT must be at least 1. If NCVT is nonzero LDVT must also be at least N.

U

U is REAL array, dimension (LDU, N)
On entry, contains a matrix which on exit has been postmultiplied by Q, dimension NRU-by-N if SQRE = 0 and NRU-by-(N+1) if SQRE = 1 (not referenced if NRU=0).

LDU

LDU is INTEGER
On entry, LDU specifies the leading dimension of U as declared in the calling (sub) program. LDU must be at least max(1, NRU) .



C

C is REAL array, dimension (LDC, NCC)
On entry, contains an N-by-NCC matrix which on exit
has been premultiplied by Q**T dimension N-by-NCC if SQRE = 0
and (N+1)-by-NCC if SQRE = 1 (not referenced if NCC=0).

LDC

LDC is INTEGER
On entry, LDC specifies the leading dimension of C as
declared in the calling (sub) program. LDC must be at
least 1. If NCC is nonzero, LDC must also be at least N.

WORK

WORK is REAL array, dimension (4*N)
Workspace. Only referenced if one of NCVT, NRU, or NCC is
nonzero, and if N is at least 2.

INFO

INFO is INTEGER
On exit, a value of 0 indicates a successful exit.
If INFO < 0, argument number -INFO is illegal.
If INFO > 0, the algorithm did not converge, and INFO
specifies how many superdiagonals did not converge.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slasdt (integer N, integer LVL, integer ND, integer, dimension(*) INODE, integer, dimension(*) NDIML, integer, dimension(*) NDIMR, integer MSUB)

SLASDT creates a tree of subproblems for bidiagonal divide and conquer. Used by sbdsdc.

Purpose:

SLASDT creates a tree of subproblems for bidiagonal divide and
conquer.

Parameters

N

N is INTEGER
On entry, the number of diagonal elements of the
bidiagonal matrix.

LVL

LVL is INTEGER
On exit, the number of levels on the computation tree.

ND

ND is INTEGER
On exit, the number of nodes on the tree.

INODE

INODE is INTEGER array, dimension (N)



On exit, centers of subproblems.

NDIML

NDIML is INTEGER array, dimension (N)
On exit, row dimensions of left children.

NDIMR

NDIMR is INTEGER array, dimension (N)
On exit, row dimensions of right children.

MSUB

MSUB is INTEGER
On entry, the maximum row dimension each subproblem at the bottom of the tree can be of.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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Contributors:

Ming Gu and Huan Ren, Computer Science Division, University of California at Berkeley, USA

subroutine slaset (character UPLO, integer M, integer N, real ALPHA, real BETA, real, dimension(lda, *) A, integer LDA)

SLASET initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

Purpose:

SLASET initializes an m-by-n matrix A to BETA on the diagonal and ALPHA on the offdiagonals.

Parameters

UPLO

UPLO is CHARACTER*1
Specifies the part of the matrix A to be set.
= 'U': Upper triangular part is set; the strictly lower triangular part of A is not changed.
= 'L': Lower triangular part is set; the strictly upper triangular part of A is not changed.
Otherwise: All of the matrix A is set.

M

M is INTEGER
The number of rows of the matrix A. M >= 0.

N

N is INTEGER
The number of columns of the matrix A. N >= 0.

ALPHA

ALPHA is REAL
The constant to which the offdiagonal elements are to be set.

BETA

BETA is REAL
The constant to which the diagonal elements are to be set.



A

A is REAL array, dimension (LDA,N)
On exit, the leading m-by-n submatrix of A is set as follows:

if UPLO = 'U', A(i,j) = ALPHA, 1<=i<=j-1, 1<=j<=n,
if UPLO = 'L', A(i,j) = ALPHA, j+1<=i<=m, 1<=j<=n,
otherwise, A(i,j) = ALPHA, 1<=i<=m, 1<=j<=n, i.ne.j,

and, for all UPLO, A(i,i) = BETA, 1<=i<=min(m,n).

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine slasr (character SIDE, character PIVOT, character DIRECT, integer M, integer N, real, dimension(*) C, real, dimension(*) S, real, dimension(lda, *) A, integer LDA)

SLASR applies a sequence of plane rotations to a general rectangular matrix.

Purpose:

SLASR applies a sequence of plane rotations to a real matrix A,
from either the left or the right.

When SIDE = 'L', the transformation takes the form

A := P*A

and when SIDE = 'R', the transformation takes the form

A := A*P**T

where P is an orthogonal matrix consisting of a sequence of z plane
rotations, with z = M when SIDE = 'L' and z = N when SIDE = 'R',
and P**T is the transpose of P.

When DIRECT = 'F' (Forward sequence), then

P = P(z-1) * ... * P(2) * P(1)

and when DIRECT = 'B' (Backward sequence), then

P = P(1) * P(2) * ... * P(z-1)

where P(k) is a plane rotation matrix defined by the 2-by-2 rotation

R(k) = (c(k) s(k))
= (-s(k) c(k)).

When PIVOT = 'V' (Variable pivot), the rotation is performed
for the plane (k,k+1), i.e., P(k) has the form



```
P(k) = (  1                              )
       (      ...                        )
       (             1                   )
       (                c(k)  s(k)       )
       (               -s(k)  c(k)       )
       (                             1    )
       (                                ... )
       (                                   1 )
```

where R(k) appears as a rank-2 modification to the identity matrix in rows and columns k and k+1.

When PIVOT = 'T' (Top pivot), the rotation is performed for the plane (1,k+1), so P(k) has the form

```
P(k) = (  c(k)                  s(k)          )
       (        1                            )
       (              ...                    )
       (                    1                )
       ( -s(k)                 c(k)          )
       (                             1       )
       (                                ...  )
       (                                   1 )
```

where R(k) appears in rows and columns 1 and k+1.

Similarly, when PIVOT = 'B' (Bottom pivot), the rotation is performed for the plane (k,z), giving P(k) the form

```
P(k) = ( 1                                  )
       (     ...                            )
       (             1                      )
       (                c(k)          s(k)  )
       (                      1             )
       (                           ...      )
       (                                1   )
       (               -s(k)          c(k)  )
```

where R(k) appears in rows and columns k and z. The rotations are performed without ever forming P(k) explicitly.

Parameters

SIDE

SIDE is CHARACTER*1
Specifies whether the plane rotation matrix P is applied to A on the left or the right.
= 'L': Left, compute A := P*A
= 'R': Right, compute A:= A*P**T

PIVOT

PIVOT is CHARACTER*1
Specifies the plane for which P(k) is a plane rotation matrix.
= 'V': Variable pivot, the plane (k,k+1)
= 'T': Top pivot, the plane (1,k+1)
= 'B': Bottom pivot, the plane (k,z)

DIRECT

DIRECT is CHARACTER*1



Specifies whether P is a forward or backward sequence of plane rotations.
= 'F': Forward, P = P(z-1)*...*P(2)*P(1)
= 'B': Backward, P = P(1)*P(2)*...*P(z-1)

M

M is INTEGER
The number of rows of the matrix A. If m <= 1, an immediate return is effected.

N

N is INTEGER
The number of columns of the matrix A. If n <= 1, an immediate return is effected.

C

C is REAL array, dimension
(M-1) if SIDE = 'L'
(N-1) if SIDE = 'R'
The cosines c(k) of the plane rotations.

S

S is REAL array, dimension
(M-1) if SIDE = 'L'
(N-1) if SIDE = 'R'
The sines s(k) of the plane rotations. The 2-by-2 plane rotation part of the matrix P(k), R(k), has the form
R(k) = (c(k) s(k))
(-s(k) c(k)).

A

A is REAL array, dimension (LDA,N)
The M-by-N matrix A. On exit, A is overwritten by P*A if SIDE = 'R' or by A*P**T if SIDE = 'L'.

LDA

LDA is INTEGER
The leading dimension of the array A. LDA >= max(1,M).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine slassq (integer N, real, dimension(*) X, integer INCX, real SCALE, real SUMSQ)

SLASSQ updates a sum of squares represented in scaled form.

Purpose:

SLASSQ returns the values scl and smsq such that

(scl**2)*smsq = x(1)**2 +...+ x(n)**2 + (scale**2)*sumsq,

where x(i) = X(1 + (i - 1)*INCX). The value of sumsq is assumed to be non-negative and scl returns the value

scl = max(scale, abs(x(i))).



scale and sumsq must be supplied in SCALE and SUMSQ and
scl and smsq are overwritten on SCALE and SUMSQ respectively.

The routine makes only one pass through the vector x.

Parameters

N

N is INTEGER
The number of elements to be used from the vector X.

X

X is REAL array, dimension (1+(N-1)*INCX)
The vector for which a scaled sum of squares is computed.
x(i) = X(1 + (i - 1)*INCX), 1 <= i <= n.

INCX

INCX is INTEGER
The increment between successive values of the vector X.
INCX > 0.

SCALE

SCALE is REAL
On entry, the value scale in the equation above.
On exit, SCALE is overwritten with scl , the scaling factor
for the sum of squares.

SUMSQ

SUMSQ is REAL
On entry, the value sumsq in the equation above.
On exit, SUMSQ is overwritten with smsq , the basic sum of
squares from which scl has been factored out.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine slasv2 (real F, real G, real H, real SSMIN, real SSMAX, real SNR, real CSR, real SNL, real CSL)

SLASV2 computes the singular value decomposition of a 2-by-2 triangular matrix.

Purpose:

SLASV2 computes the singular value decomposition of a 2-by-2
triangular matrix
[F G]
[0 H].
On return, abs(SSMAX) is the larger singular value, abs(SSMIN) is the
smaller singular value, and (CSL,SNL) and (CSR,SNR) are the left and
right singular vectors for abs(SSMAX), giving the decomposition

[CSL SNL] [F G] [CSR -SNR] = [SSMAX 0]
[-SNL CSL] [0 H] [SNR CSR] [0 SSMIN].

Parameters

F

F is REAL



The (1,1) element of the 2-by-2 matrix.

G

G is REAL
The (1,2) element of the 2-by-2 matrix.

H

H is REAL
The (2,2) element of the 2-by-2 matrix.

SSMIN

SSMIN is REAL
abs(SSMIN) is the smaller singular value.

SSMAX

SSMAX is REAL
abs(SSMAX) is the larger singular value.

SNL

SNL is REAL

CSL

CSL is REAL
The vector (CSL, SNL) is a unit left singular vector for the singular value abs(SSMAX).

SNR

SNR is REAL

CSR

CSR is REAL
The vector (CSR, SNR) is a unit right singular vector for the singular value abs(SSMAX).

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Further Details:

Any input parameter may be aliased with any output parameter.

Barring over/underflow and assuming a guard digit in subtraction, all output quantities are correct to within a few units in the last place (ulps).

In IEEE arithmetic, the code works correctly if one matrix element is infinite.

Overflow will not occur unless the largest singular value itself overflows or is within a few ulps of overflow. (On machines with partial overflow, like the Cray, overflow may occur if the largest singular value is within a factor of 2 of overflow.)

Underflow is harmless if underflow is gradual. Otherwise, results



may correspond to a matrix modified by perturbations of size near
the underflow threshold.

subroutine xerbla (character*(*) SRNAME, integer INFO)

XERBLA

Purpose:

XERBLA is an error handler for the LAPACK routines.
It is called by an LAPACK routine if an input parameter has an
invalid value. A message is printed and execution stops.

Installers may consider modifying the STOP statement in order to
call system-specific exception-handling facilities.

Parameters

SRNAME

SRNAME is CHARACTER*(*)
The name of the routine which called XERBLA.

INFO

INFO is INTEGER
The position of the invalid parameter in the parameter list
of the calling routine.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

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subroutine xerbla_array (character(1), dimension(srname_len) SRNAME_ARRAY, integer SRNAME_LEN, integer INFO)

XERBLA_ARRAY

Purpose:

XERBLA_ARRAY assists other languages in calling XERBLA, the LAPACK
and BLAS error handler. Rather than taking a Fortran string argument
as the function's name, XERBLA_ARRAY takes an array of single
characters along with the array's length. XERBLA_ARRAY then copies
up to 32 characters of that array into a Fortran string and passes
that to XERBLA. If called with a non-positive SRNAME_LEN,
XERBLA_ARRAY will call XERBLA with a string of all blank characters.

Say some macro or other device makes XERBLA_ARRAY available to C99
by a name lapack_xerbla and with a common Fortran calling convention.
Then a C99 program could invoke XERBLA via:

```
{
  int flen = strlen(__func__);
  lapack_xerbla(__func__, &flen, &info);
}
```

Providing XERBLA_ARRAY is not necessary for intercepting LAPACK
errors. XERBLA_ARRAY calls XERBLA.

Parameters

SRNAME_ARRAY



SRNAME_ARRAY is CHARACTER(1) array, dimension (SRNAME_LEN)
The name of the routine which called XERBLA_ARRAY.

SRNAME_LEN

SRNAME_LEN is INTEGER
The length of the name in SRNAME_ARRAY.

INFO

INFO is INTEGER
The position of the invalid parameter in the parameter list
of the calling routine.

Author

Univ. of Tennessee

Univ. of California Berkeley

Univ. of Colorado Denver

NAG Ltd.

Date

December 2016

Author

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